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AIR FORCE AFR 88-19, Volume 6

TECHNICAL MANUAL

ARCTIC AND SUBARCTIC CONSTRUCTION CALCULATION METHODS FOR DETERMINATION OF DEPTHS OF FREEZE AND THAW IN SOILS

DEPARTMENTS OF THE ARMY AND THE AIR FORCE

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ARCTIC AND SUBARCTIC CONSTRUCTION—CALCULATION METHODS FOR DETERMINATION OF DEPTHS OF FREEZE AND THAW IN SOILS

*This manual supercedes TM 5-852-6/AFM 88-19, Chap. 6, dated January 1966.

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CHAPTER 1

GENERAL

1-1. Purpose and scope.

This manual contains criteria and methods for calculating the depths of freeze and thaw in soils, with consideration of the effects of other adjacent materials, for the design of military facilities in seasonal frost, arctic and subarctic regions. The contents are applicable to both Army and Air Force construction in arctic, subarctic and seasonal frost areas. The data presented in this manual relate to arctic and subarctic facility design presented in the other manuals of the Arctic and Subarctic Construction series.

1-2. References and symbols.

Appendix C lists the references for this manual; appendix A contains a list of symbols.

1-3. Background.

a. The depths to which soils may freeze and thaw is very important in the design of pavements, structures and utilities in areas of seasonal frost and permafrost. Methods of calculating such depths, based on heat-transfer principles, are presented here. For derivation of basic equations, and the underlying theory, see appendix C and the bibliography.

b. Heat transfer in soils involving phase change of pore water is an extremely complex process and many problems defy rigorous mathematical treatment. The methods presented here are simplified procedures developed for the solution of engineering design problems.

c. Several assumptions have been made in developing practical methods of calculating depths of freeze or thaw in soils. It is assumed that each layer of material is homogeneous and isotropic, and that the average thermal properties of frozen and unfrozen soils are applicable. Unless specific data are available, it is also assumed that all soil water is converted to ice, or all ice is converted to water, at a temperature of 32°F. This latter assumption is substantially correct for coarse-grained soils but only partially true for fine-grained soils.

d. The services of the U.S. Army Cold Regions Research and Engineering Laboratory (USACRREL), Hanover, New Hampshire, are available to assist in the development of solutions for heat-flow problems in soils.

CHAPTER 2

DEFINITIONS AND THERMAL PROPERTIES

2-1. Definitions.

Definitions of certain specialized terms applicable to arctic and subarctic regions are contained in TM 5-852/AFR 88-19, Volume 1. Following are additional terms used specifically in heat-transfer calculations.

a. Thermal conductivity, K. The quantity of heat flow in a unit time through a unit area of a substance caused by a unit thermal gradient.

b. Specific heat, c. The quantity of heat absorbed (or given up) by a unit weight of a substance when its temperature is increased (or decreased) by 1 degree Fahrenheit (°F) divided by the quantity of heat absorbed (or given up) by a unit weight of water when its temperature is increased (or decreased) by 1°F.

c. Volumetric heat capacity, C. The quantity of heat required to change the temperature of a unit volume by 1°F.

—*For unfrozen soils,*

$$C_u = \gamma_d \left(c + 1.0 \frac{w}{100}\right). \qquad \text{(eq 2-1)}$$

—*For frozen soils,*

$$C_f = \gamma_d \left(c + 0.5 \frac{w}{100}\right). \qquad \text{(eq 2-2)}$$

—*Average values for most soils,*

$$C_{avg} = \gamma_d \left(c + 0.75 \frac{w}{100}\right). \qquad \text{(eq 2-3)}$$

where c = specific heat of the soil solids (0.17 for most soils)

γ_d = dry unit weight of soil

w = water content of soil in percent of dry weight.

d. Volumetric latent heat of fusion, L. The quantity of heat required to melt the ice (or freeze the water) in a unit volume of soil without a change in temperature—in British thermal units (Btu) per cubic foot (ft^3):

$$L = 144 \gamma_d \frac{w}{100} \qquad \text{(eq 2-4)}$$

e. thermal resistance, R. The reciprocal of the time rate of heat flow through a unit area of a soil layer of given thickness d per unit temperature difference:

$$R = \frac{d}{K} \qquad \text{(eq 2-5)}$$

f. Thermal diffusivity, a. An indicator of how easily a material will undergo temperature change:

$$a = \frac{K}{C} \qquad \text{(eq 2-6)}$$

g. Thermal ratio, α.

$$\alpha = \frac{v_o}{v_s} \qquad \text{(eq 2-7)}$$

where

v_o = absolute value of the difference between the mean annual temperature below the ground surface and 32°F.

v_s = one of two possible meanings, depending on the problem being studied:

(1) $v_s = nF/t$ (or nI/t)

where

n = conversion factor from air index to surface index

F = air freezing index

I = air thawing index

t = length of freezing (or thawing) season.

(2) v_s = absolute value of the difference between the mean annual ground surface temperature and 32°F.

(3) In the first case, v_s is useful for computing the seasonal depth of freeze or thaw. In the second case, it is useful in computing multiyear freeze or thaw depths that may develop as a result of some long-term change in the heat balance at the ground surface.

h. Fusion parameter, μ.

$$\mu = \frac{C}{L} v_s \qquad \text{(eq 2-8)}$$

where v_s has the two possible meanings noted above.

i. "Lambda" coefficient, λ. A factor allowing for heat capacity and initial temperature of the ground (see fig. 3-1).

$$\lambda = f(\alpha,\mu) \qquad \text{(eq 2-9)}$$

j. Thermal regime. The temperature pattern existing in a soil body in relation to seasonal variations.

k. British thermal unit, Btu. The quantity of heat required to raise the temperature of 1 pound (lb) of water 1°F at about 40°F.

2-2. Thermal properties of soils and other construction materials.

a. The basic thermal properties of soils and other construction materials used to calculate depths of freeze and thaw are specific heat, thermal conductivity and volumetric latent heat of fusion. Other terms used in heat-flow calculations are derived from these data and the elements of weight, length, temperature and time.

b. the specific heat of most dry soils near the freezing point may be assumed to be constant at the value of 0.17 Btu/lb °F. Specific heats of various materials are given in table 2-1; see the *ASHRAE Guide and Data Book* of the American

*Table 2-1. Specific heat values of various materials**
(U.S. Army Corps of Engineers).

Material	Temperature (°F)	Specific heat (Btu/lb °F)
Aluminum	-27.4	0.20
Asbestos fibers	--	0.25
Concrete (avg. stone)	--	0.20
Concrete (dams)	--	0.22
Copper	44.6	0.20
Corkboard	91	0.43
	-19	0.29
Cork, granulated	--	0.42
Fiberglas board	111	0.24
	-22	0.19
Foamglas	-20	0.16
Glass block, expanded	112	0.18
Glass sheets	--	0.20
Glass wool	--	0.16
Ice	32	0.48
Iron (alpha)	44.6	0.11
Masonry	--	0.22
Mineral wool	--	0.22
Perlite, expanded	--	0.22
Polystyrene, cellular foam	--	0.27
Polyurethane foam	--	0.25
Sawdust	--	0.60
Snow	--	0.50
Steel	--	0.12
Straw	--	0.35
Water	--	1.00
Woods (avg.)	68	0.33
Woods fiberboard	148	0.34

* Specific heat values shown to nearest 0.01. Average values listed where temperature is not shown.

Society of Heating and Air Conditioning Engineers for the specific heat values of common materials.

c. The thermal conductivity of soils is dependent upon a number of factors: density; moisture content; particle shape; temperature; solid, liquid and vapor constituents; and the state of the pore water, whether frozen or unfrozen. Average values, expressed in Btu/ft hour °F, for frozen and unfrozen granular soils, silts and clays should be read from figures 2-1 through 2-4. The charts for sands and gravels are applicable when the silt and clay content together make up less than 20% of the soil solids. The charts for silt and clay are applicable when that fraction is at least 50%. For intermediate silt-clay fractions, it is recommended that the simple average of the values for the two sets of charts be used. In all cases, the error in the thermal conductivity estimates may be ±25%, and even higher when the percentage of quartz grains in the soil is exceptionally high or low. Figures 2-5 and 2-6 present estimates of the average thermal conductivity of frozen and unfrozen peat. An excellent source of data for dry construction materials is the *ASHRAE Guide and Data Book*. Thermal conductivity values for a number of common construction materials are listed in table 2-2.

d. The latent heat of fusion is the amount of heat required to cause a phase change in soil moisture. This amount of heat does not change the temperature of the system when freezing or thawing takes place. The gravimetric latent heat of fusion of water is assumed to be 144 Btu/lb. The amount of heat energy required to convert 1 ft of water to ice is (144 × 62.4 =) 9000 Btu/ft^3 and to change 1 ft^3 of ice to water is (144 × 0.917 × 62.4 =) 8240 Btu/ft^3. (Note: The density of water 62.4 lb/ft^3 and that of pure ice is 57.2 lb.ft^3).

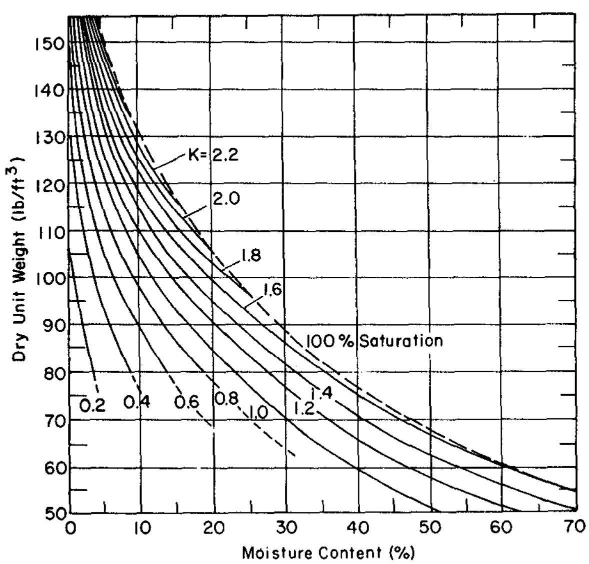
e. Figures 2-7 and 2-8 may be used to determine the average volumetric heat capacity and volumetric latent heat of fusion, respectively, of moist soils.

f. The following *example* illustrates the significance of latent heat of fusion relative to the volumetric heat capacity for a moist soil. Assume a soil having a dry unit weight of 120 lb/ft^3 and a water content of 15 percent. Its volumetric latent heat of fusion, L, is (144 × 120 × 0.15 =) 2592 Btu/ft^3 and its average volumetric heat capacity, C, is (120[0.17 + 0.75 × 0.15] =) 33.9 Btu/ft^3 °F. The quantity of heat required to change the phase of pore water in 1 ft^3 of this soil at 32°F is the same as that required to cause a temperature change of (2592/33.9 =) 76.4°F when a phase change is not involved.

2-3. Fundamental considerations.

a. Theoretical basis. The freezing or thawing of soils is the result of removing or adding heat to an existing soil mass. The movement of heat is always in the direction of lower temperature. The time rate of change of heat content depends on the temperature differential in the direction of heat flow and on the thermal properties of the soil.

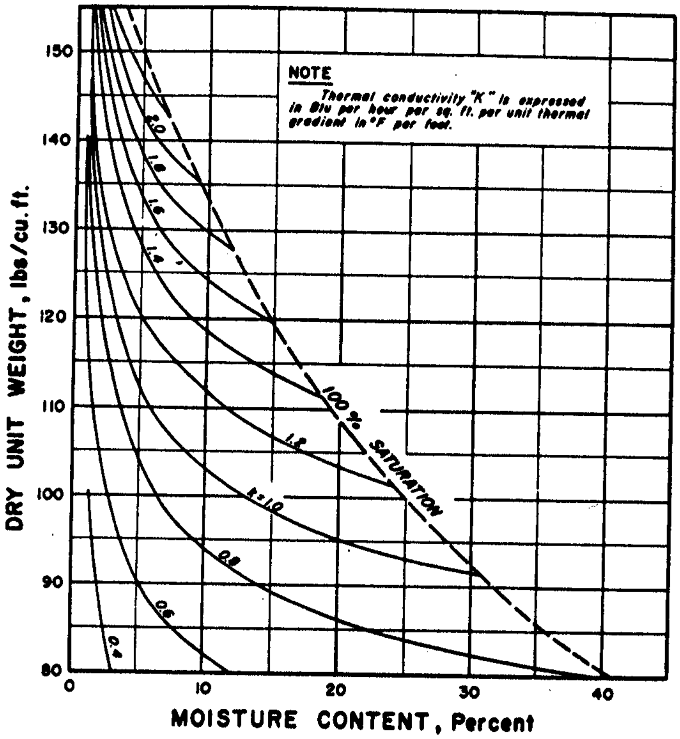
b. Physical factors and data required. Calculation of the depth of freeze or thaw is based on knowledge of the physical and thermal properties of the soil in the profile, the existing thermal regime, and the nature and duration of boundary conditions



Thermal conductivity K is expressed in Btu per hour per square foot per unit thermal gradient in °F per foot.
Dashed line represents extrapolation.

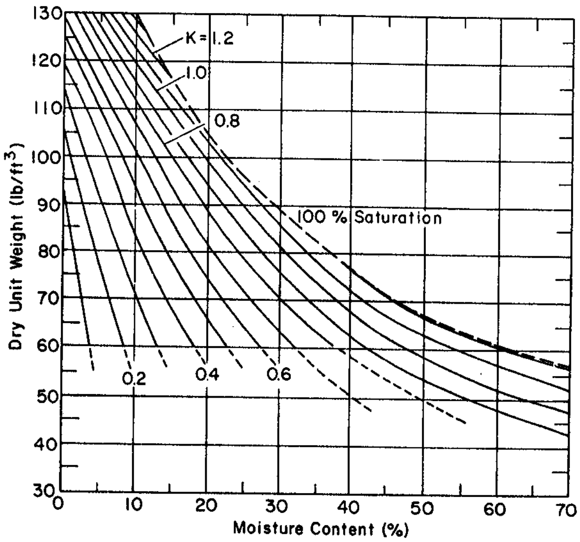
(U.S. Army Corps of Engineers)

Figure 2-1. Average thermal conductivity for sands and gravels, frozen.



(U.S. Army Corps of Engineers)

Figure 2-2. Average thermal conductivity for sands and gravels, unfrozen.



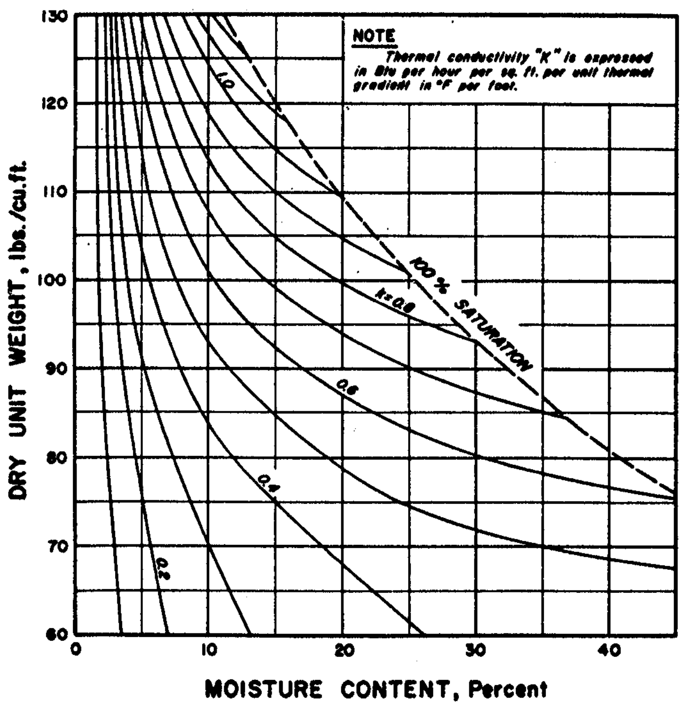
γ_d	w: 90	120	100% saturated
50	N/A	N/A	1.20
40	1.00	N/A	1.23
30		0.98	1.26
20		0.61	1.26
pure ice			1.26

Thermal conductivity K is expressed in Btu per hour per square foot per unit thermal gradient in °F per foot.

Dashed line represents extrapolation.

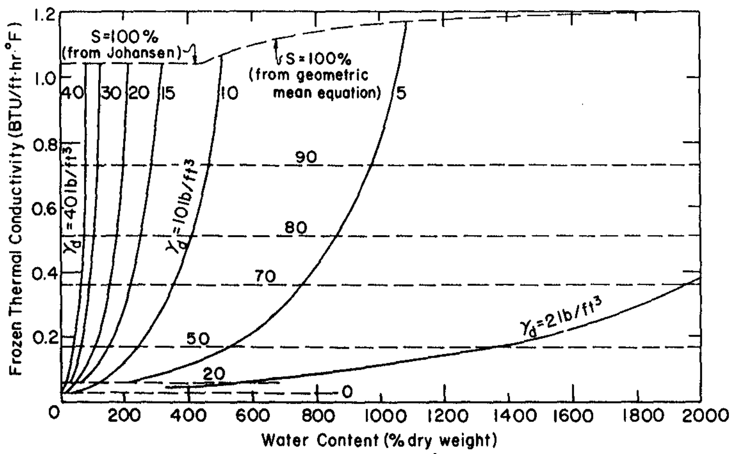
(U.S. Army Corps of Engineers)

Figure 2-3. Average thermal conductivity for silt and clay soils, frozen.



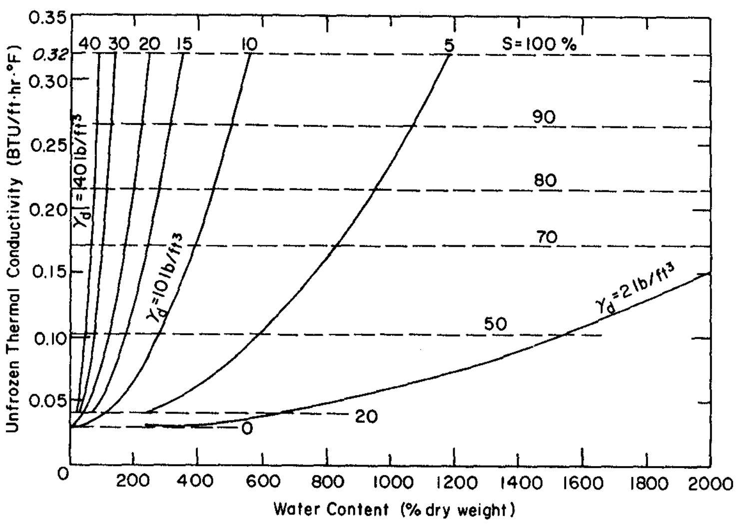
(U.S. Army Corps of Engineers)

Figure 2-4. Average thermal conductivity for silt and clay soils, unfrozen.



(U.S. Army Corps of Engineers)

Figure 2-5. Average thermal conductivity for peat, frozen.



(U.S. Army Corps of Engineers)

Figure 2-6. Average thermal conductivity for peat, unfrozen.

causing a change in the thermal regime. Data pertinent to the soil profile include grain-size distribution, classification, density, water or ice content, and temperature of each soil stratum. Knowledge of the thermal properties of all the materials in the heat flow path is also required. Measured or assumed temperatures within the soil mass determine the initial conditions. If surface temperatures can be assumed to be spatially uniform and the thermal influence of any buried structures can be considered negligible, one-dimensional heat flow may be assumed, thereby simplifying the problem and its solution.

2-4. Freezing and thawing indexes.

a. Physical concept and quantitative measurement. The penetration of freezing or thawing temperatures into soil partly depends on the magnitude and duration of the temperature differential at the air-ground interface. The magnitude of the temperature differential is expressed as the number of degrees that the temperature in the air

Table 2-2. Thermal properties of construction materials (U.S. Army Corps of Engineers).

Type of material	*Description*	*Unit weight (lb/ft³)*	*k conductivity** *(Btu/ft²·hr·°F per in.)*	*K conductivity (Btu/ft·hr·°F)*
Asphalt paving mixture	Mix with 6% by weight cut-back asphalt	*138*	10.3	0.86
Concrete	With sand and gravel or stone aggregate (oven-dried)	*140*	9.0	0.75
	With sand and gravel or stone aggregate (not dried)	*140*	12.0	1.00
	With lightweight aggregates, including expanded shale, clay or slate; expanded slags; cinders; pumice; perlite; vermiculite; also cellular concretes.	*120*	5.2	0.43
		100	3.6	0.30
		80	2.5	0.21
		60	1.7	0.14
		40	1.15	0.096
		30	0.90	0.075
		20	0.70	0.058
Wood	Maple, oak and similar hardwoods	*45*	*1.10*	0.092
	Fir, pine and similar softwoods	*32*	0.80	0.067
Building boards	Asbestos-cement board	*120*	4.0	0.33
	Plywood	*34*	0.80	0.067
	Wood fiberboard, laminated or homogeneous	*26,33*	0.42,0.55	0.035,0.046
	Wood fiber-hardboard type	*65*	1.40	0.12
Blanket and batt insulation	Mineral wool, fibrous form, processed from rock, slag, or glass	*1.5-4.0*	0.27	0.022
	Wood fiber	*3.2-3.6*	0.25	0.021
Board and slab insulation	Cellular glass	*9.0*	†0.39	0.032
	Corkboard (without added binder)	*6.5-8.0*	†0.27	0.022
	Glass fiber	*9.5-11.0*	0.25	0.021
	Wood or cane fiber-interior finish (plank, tile, lath)	*15.0*	0.35	0.029
	Expanded polystyrene	*1.6*	0.29	0.024
	Expanded ureaformaldehyde	*1.0*	0.25	--
	Expanded perlite	*9.5-11.5*	0.34	--
	Polyurethane foam	*1.5-3.0*	0.17	--
	Mineral wool with resin binder	*15.0*	**0.28	0.023
	Mineral wool with asphalt binder	*15.0*	**0.31	0.026

Table 2-2. Thermal properties of construction materials, Continued

Type of material	Description	Unit weight (lb/ft^3)	k conductivity* (Btu/ft^2·hr·°F per in.)	K conductivity (Btu/ft·hr·°F)
Loose fill insulation	Cork, granulated	5-12	0.25-0.36	--
	Expanded perlite	3-4	0.28	--
	Mineral wool (glass, slag, or rock)	2-5	0.30	0.025
	Sawdust or shavings	8-15	0.45	0.037
	Straw	7-8	0.32	
	Vermiculite (expanded)	7.0-8.2	0.48	0.040
	Wood fiber: redwood, hemlock, or fir	2.0-3.5	0.30	0.025
Miscellaneous	Aluminum	168	1416	118
	Copper	549	2640	220
	Ductile iron	468	360	30
	Glass	164	5.5	0.46
	Ice	57	**15.4	1.28
	Snow, new, loose	5.3	0.6	0.05
	Snow on ground	18.7	1.56	0.13
	Snow, drifted and compacted	31.2	4.8	0.40
	Steel	487	310	25.8
	Water, average	62.4	4.2	0.35

* Values for k are for dry building materials at a mean temperature of 75° F except as noted; wet conditions will adversely affect values of many of these materials.

† Mean temperature of 60° F.

**Mean temperature of 32° F.

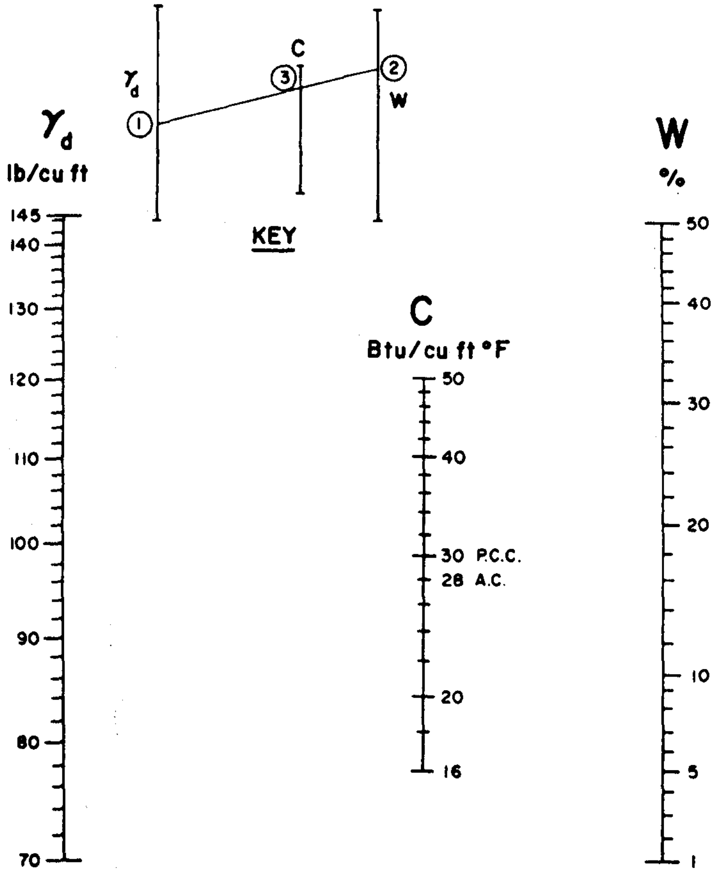
or at the ground surface is above (positive) or below (negative) 32°F, the assumed freezing point of water. The duration is expressed in days.

b. Air freezing index (F) and air thawing index (I). The air freezing and air thawing indexes, as defined in TM 5-852-1/AFR 88-19, Volume 1, may be determined by the following methods.

(1) Summation of degree-days of freeze and thaw from average daily temperatures. If T_1 is the maximum daily air temperature and T_2 is the minimum daily air temperature, the average daily air temperature $\overline{T}$, may be taken as $1/2\,(T_1 + T_2)$, and the number of degree-days for the day is $(\overline{T} - 32)$. The summation of the degree-days for a freezing or thawing season gives the air freezing or air thawing index. Table 2-3 illustrates the method used to obtain the summation of degree-days for a 1-week period, assuming that -456 degree-days had been accumulated since freezing began. An average daily temperature based on hourly temperatures would be slightly more accurate, but such precision is not usually warranted. The negative sign, indicating freezing degree-days, is usually omitted.

(2) Calculation from average monthly temperatures. The freezing or thawing index may be calculated by determining the area between the 32°F line and the curve of average monthly temperature and time, taken over the appropriate season. The area may be determined by planimeter or a simple approximation rule (Simpson's rule, midordinate rule, etc.). The areas are expressed in units of degree-days, resulting in a summation of degree-days or a freezing or thawing index. For an example refer to figure 2-9, a plot of the monthly average temperatures at Fairbanks, Alaska, from September 1949 to October 1950. Determination of areas by planimeter gave a freezing index of -5240 degree-days and a thawing index of +3420 degree-days. The use of Simpson's rule gave a freezing index of -5390 and a thawing index of +3460 degree-days. Either pair of indices is adequate for computations.

c. Surface-freezing and surface-thawing indexes. For determining the heat flow within the soil, it is neces-



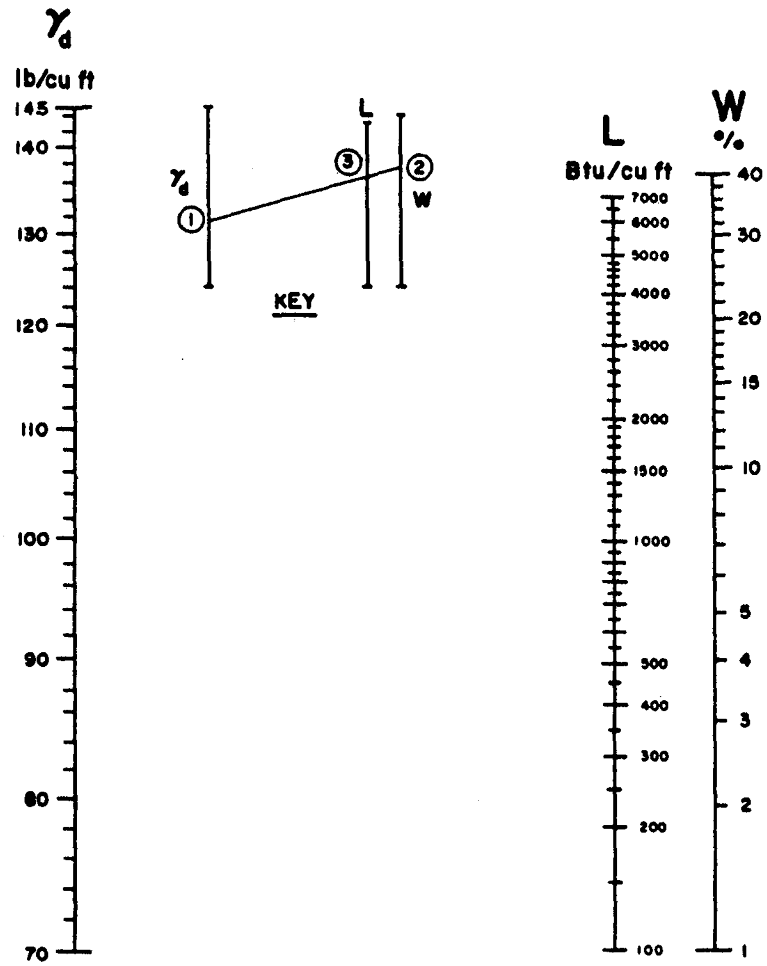
NOTE: Specific heat of soil solids assumed to be 0.17 Btu/lb.°F

(U.S. Army Corps of Engineers)

Figure 2-7. Average volumetric heat capacity for soils.

sary to determine or estimate the temperature condition at the ground surface. Since air temperatures are generally available and surface temperatures are not, a correlation between these temperatures helps establish the thermal boundary condition at the ground surface. The combined effects of radiative, convective and conductive heat exchange at the air-ground interface often must be considered in determining surface temperature.

d. Correlation of air and surface indexes. No simple correlation exists between air and surface indexes. The difference between air and surface temperatures at any specific time is influenced by latitude, cloud cover, time of year, time of day, atmospheric humidity and stability, wind speed, snow cover and ground surface characteristics, and subsurface thermal properties. Heat balance algorithms that approximate many of these interrelationships exist but they are often unwieldly to use and the inputs are often difficult to characterize. It is recommended that the ratio of surface index to air index, designated as the "n-factor," be used to represent a monthly or seasonal correlation. Reliable determination of the n-factor for a specific location requires concurrent observations of air and surface temperatures throughout a number of complete freezing and thawing seasons plus anticipation of future changes to conditions existing during the period of measurement. Such determination is often not feasible, so n-factors must generally be estimated conservatively from n-factors tabulated for other, preferably similar, sites.



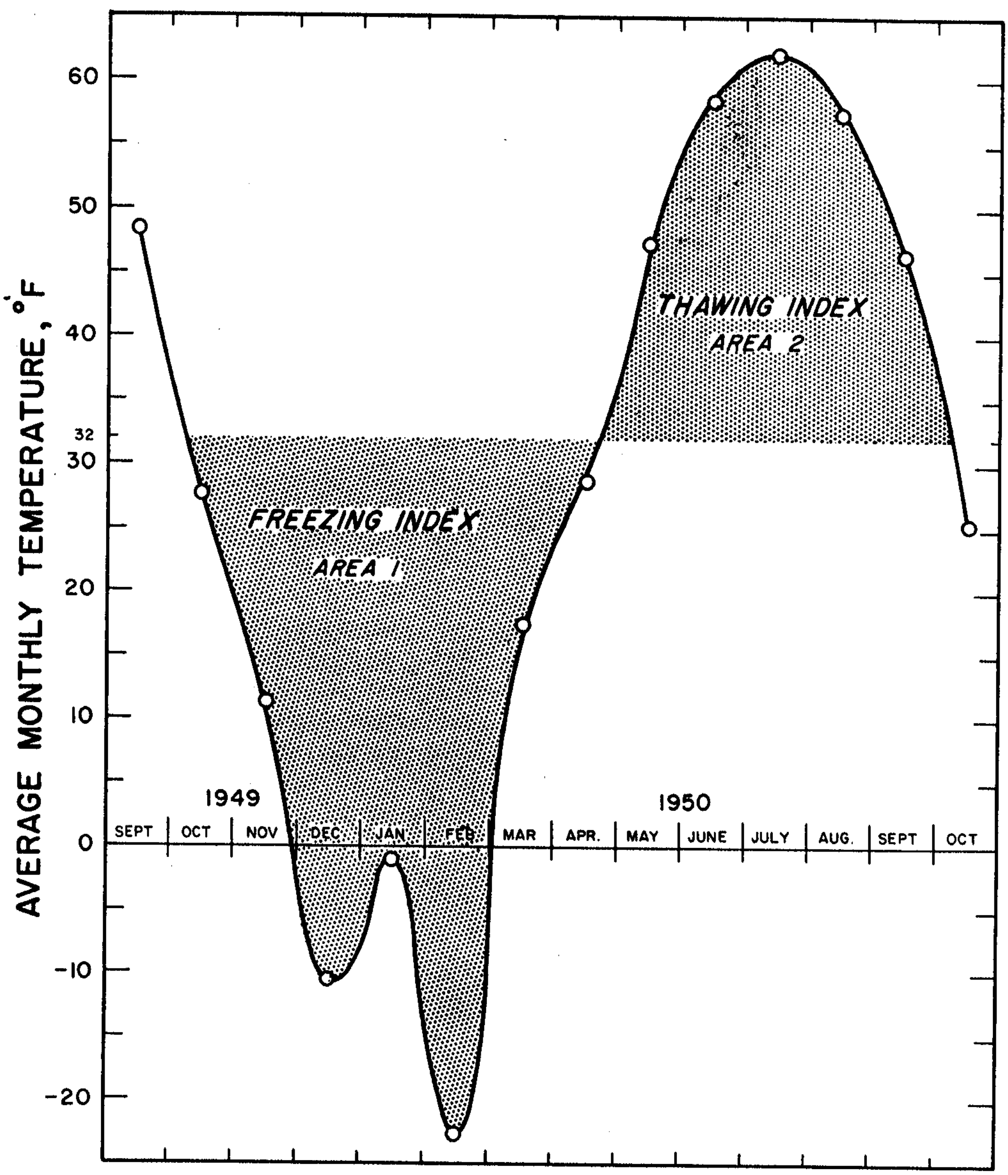
(U.S. Army Corps of Engineers)

Figure 2-8. Volumetric latent heat for soils.

Table 2-3. Calculation of cumulative degree-days (U.S. Army Corps of Engineers).

Day	Temperature (°F) Maximum	Minimum	Average	Degree-days per day	Cumulative degree-days
1	29	1	15	-17	-473*
2	9	-11	- 1	-33	-506
3	10	- 8	1	-31	-537
4	15	- 1	7	-25	-562
5	30	16	23	- 9	-571
6	38	30	34	2	-569
7	30	18	24	- 8	-577

*Prior accumulation of -456 degree-days assumed.



(U.S. Army Corps of Engineers)

Figure 2-9. Average monthly temperatures versus time at Fairbanks, Alaska.

(1) *Freezing conditions.* The n-factor is very significant in analytical ground studies. It generally increases with wind speed. Snow cover reflects a large part of incoming solar radiation resulting in higher freezing indexes at the snow surface, but its insulating effect can greatly reduce the freezing index at the ground surface. The effects of turf or an organic ground cover on the heat flow processes at the air-ground interface are extremely variable and difficult to evaluate. On the basis of observations and studies made to date, the n-factors given for average conditions in table 2-4 should be used to convert the air freezing index to the surface freezing index in the absence of specific measurements at the site of planned construction.

(2) *Thawing conditions.* The n-factor for thawing conditions is affected by the same factors as those for freezing conditions. It is the ratio of surface degree-days of thaw (degrees above 32°F) to air degree-days of thaw.

Incoming shortwave radiation may introduce heat into the surface to an extent that the surface becomes a source of heat conducted not only downward but upward into the air. In such a case the n-factor may become significantly larger than 1.0. The effect of latitude is not particularly significant in arctic and subarctic areas, but consideration should be given to the effect of wind speed. Recommended curves for n-factors versus wind speed for portland-cement-concrete and bituminous-concrete pavements are shown in figure 2-10 and are based on field studies conducted in Alaska and Greenland. The n-factors given for average conditions in table 2-4 should be used to convert air thawing indexes to surface thawing indexes in the absence of specific measurements at the planned construction sites.

e. Design indexes. For design of permanent pavements, the design freezing (or thawing) index should be the average air freezing (or thawing) index of the three coldest winters (or warmest summers) in the latest 30 years of

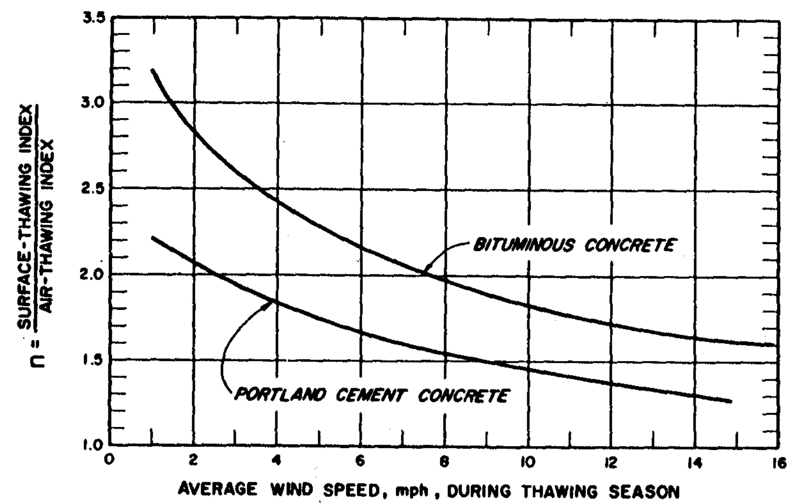
Table 2-4. n-factors for freeze and thaw (ratio of surface index to air index) (U.S. Army Corps of Engineers).

Type of surface*	For freezing conditions	For thawing conditions
Snow surface	1.0	--
Portland-cement concrete	0.75	1.5
Bituminous pavement	0.7	1.6 to 2†
Bare soil	0.7	1.4 to 2†
Shaded surface	0.9	1.0
Turf	0.5	0.8
Tree-covered	0.3**	0.4

* Surface exposed directly to sun or air without any overlying dust, soil, snow or ice, except as noted otherwise, and with no building heat involved.

† Use lowest value except in extremely high latitudes or at high elevations where a major portion of summer heating is from solar radiation.

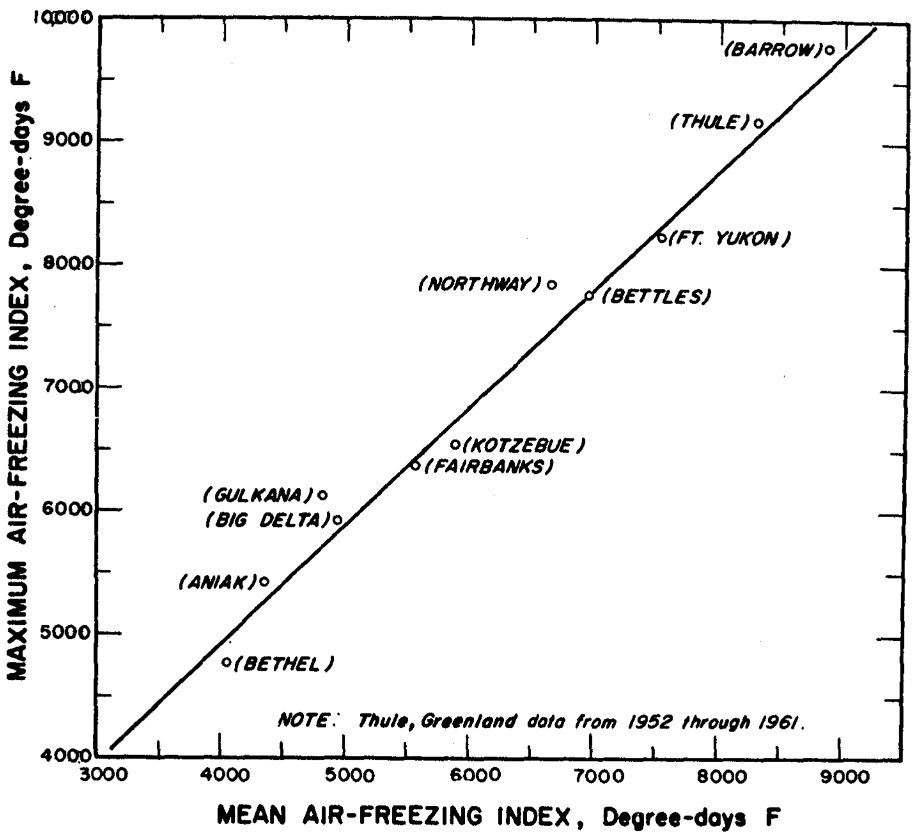
** Data from Fairbanks, Alaska, for single season with snow cover permitted to accumulate naturally.



(U.S. Army Corps of Engineers)

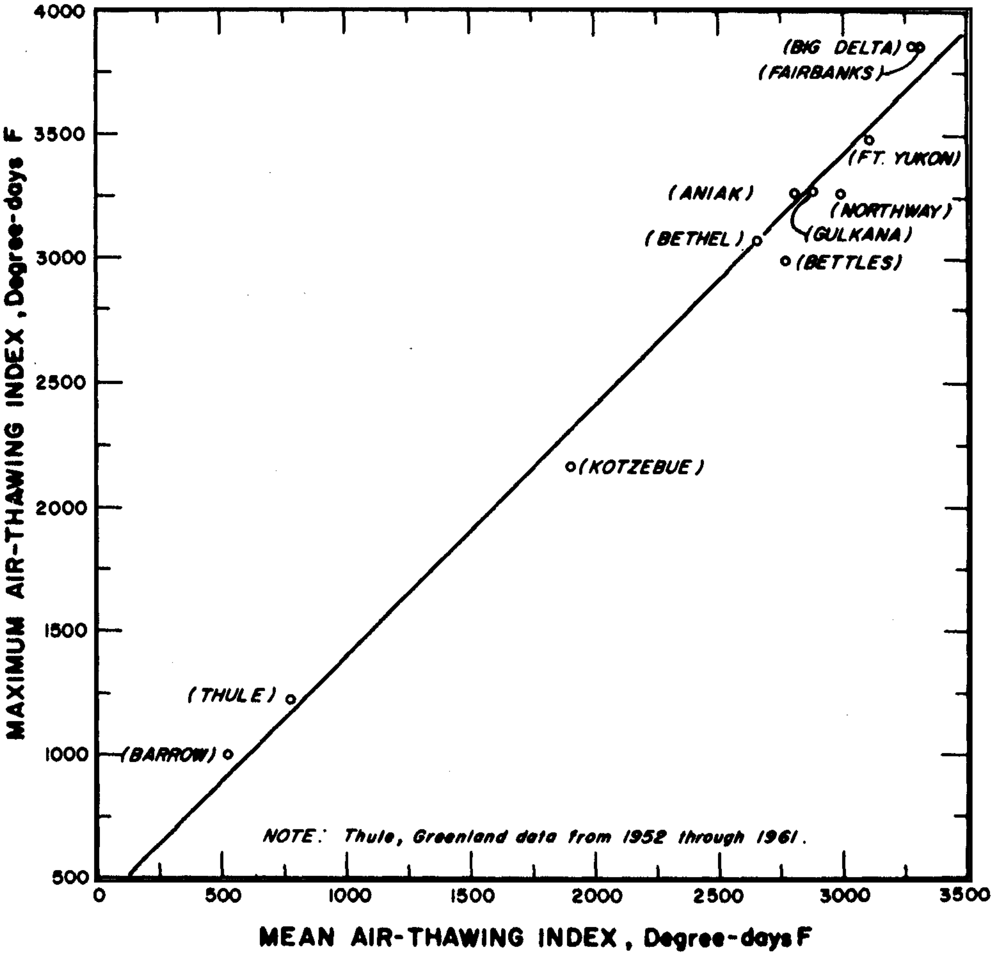
Figure 2-10. Relationship between wind speed and n-factor during thawing season.

record. If 30 years of record are not available, the air freezing (or thawing) index for the coldest winter (or warmest summer) in the latest 10-year period may be used. For design of foundations for average permanent structures, the design freezing (or thawing) index should be computed for the coldest (or warmest) winter in 30 years of record or should be estimated to correspond with this frequency if the number of years of record is limited. Periods of record used should be the latest available. To avoid the necessity for adopting a new and only slightly different freezing (or thawing) index each year, the design index at a site with continuing construction need not be changed more often than once in 5 years unless the more recent temperature records indicate a significant change. The distribution of design freezing and thawing index values in North America is presented in TM 5-852-1/AFR 88-19, Volume 1. The relatively linear relationship between recorded maximum indexes and mean freezing and thawing indexes shown in figures 2-11 and 2-12 may be used in conjunction with distribution of mean and freezing and thawing indexes in TM 5-852-1/AFR 88-19, Volume 1 to determine the design index values for arctic and subarctic regions.



(U.S. Army Corps of Engineers)

Figure 2-11. Relationship between mean freezing index and maximum freezing index for 10 years of record, 1953-1962 (arctic and subarctic regions).



(U.S. Army Corps of Engineers)

Figure 2-12. Relationship between mean thawing index and maximum thawing index for 10 years of record, 1953-1962 (arctic and subarctic regions).

CHAPTER 3

ONE-DIMENSIONAL LINEAR AND PERIODIC HEAT FLOW

3-1. Thermal regime.

The seasonal depths of frost and thaw penetration in soils depends upon the thermal properties of the soil mass, the surface temperature (upper boundary condition) and the thermal regime of the soil at the start of the freezing or thawing season. Many methods are available to estimate frost and thaw penetration depths and surface temperatures. Some of these are summarized in appendix B. This chapter concentrates on some techniques that require only relatively simple hand calculations. For the computational methods discussed below, the initial ground temperature is assumed to uniformly equal the mean annual air temperature of the particular site under consideration. The upper boundary condition is represented by the surface freezing (or thawing) index.

3-2. Modified Berggren equation.

a. The depth to which 32°F temperatures will penetrate into the soil mass is based upon the "modified" Berggren equation, expressed as:

$$X = \lambda \sqrt{\frac{48\,K\,nF}{L}}$$

or (eq 3-1)

$$X = \lambda \sqrt{\frac{48\,K\,nI}{L}}$$

where

X = depth of freeze or thaw (ft)

K = thermal conductivity of soil (Btu/ft hr °F)

L = volumetric latent heat of fusion (Btu/ft^3)

n = conversion factor from air index to surface index (dimensionless)

F = air freezing index (°F-days)

I = air thawing index (°F-days)

λ = coefficient that considers the effect of temperature changes in the soil mass (dimensionless).

The λ coefficient is a function of the freezing (or thawing) index, the mean annual temperature of the site, and the thermal properties of the soil. Freeze and thaw of low-moisture-content soils in the lower latitudes is greatly influenced by this coefficient. It is determined by two factors: the thermal ratio α and the fusion parameter μ. These have been defined in paragraph 2-1. Figure 3-1 shows λ as a function of α and μ.

b. A complete development of this equation and a discussion of the necessary assumptions and simplifications made during its development are not presented here. A few of the more important assumptions and some of the equation limitations are discussed below. The assumptions and limitations apply regardless of whether the equation is used to determine the depth of freeze or the depth of thaw.

(1) *Assumptions.* The mathematical model assumes one-dimensional heat flow with the entire soil mass at its mean annual temperature (MAT) prior to the start of the freezing season. It assumes that when the freezing season starts, the surface temperature changes suddenly (as a step function) from the mean annual temperature to a temperature v_s degrees below freezing and that it remains at this new temperature throughout the entire freezing season. Latent heat affects the model by acting as a heat sink at the moving frost line, and the model assumes that the soil freezes at a temperature of 32°F.

(2) *Limitations.* The modified Berggren equation is able to determine frost penetration in areas where the ground below a depth of several feet

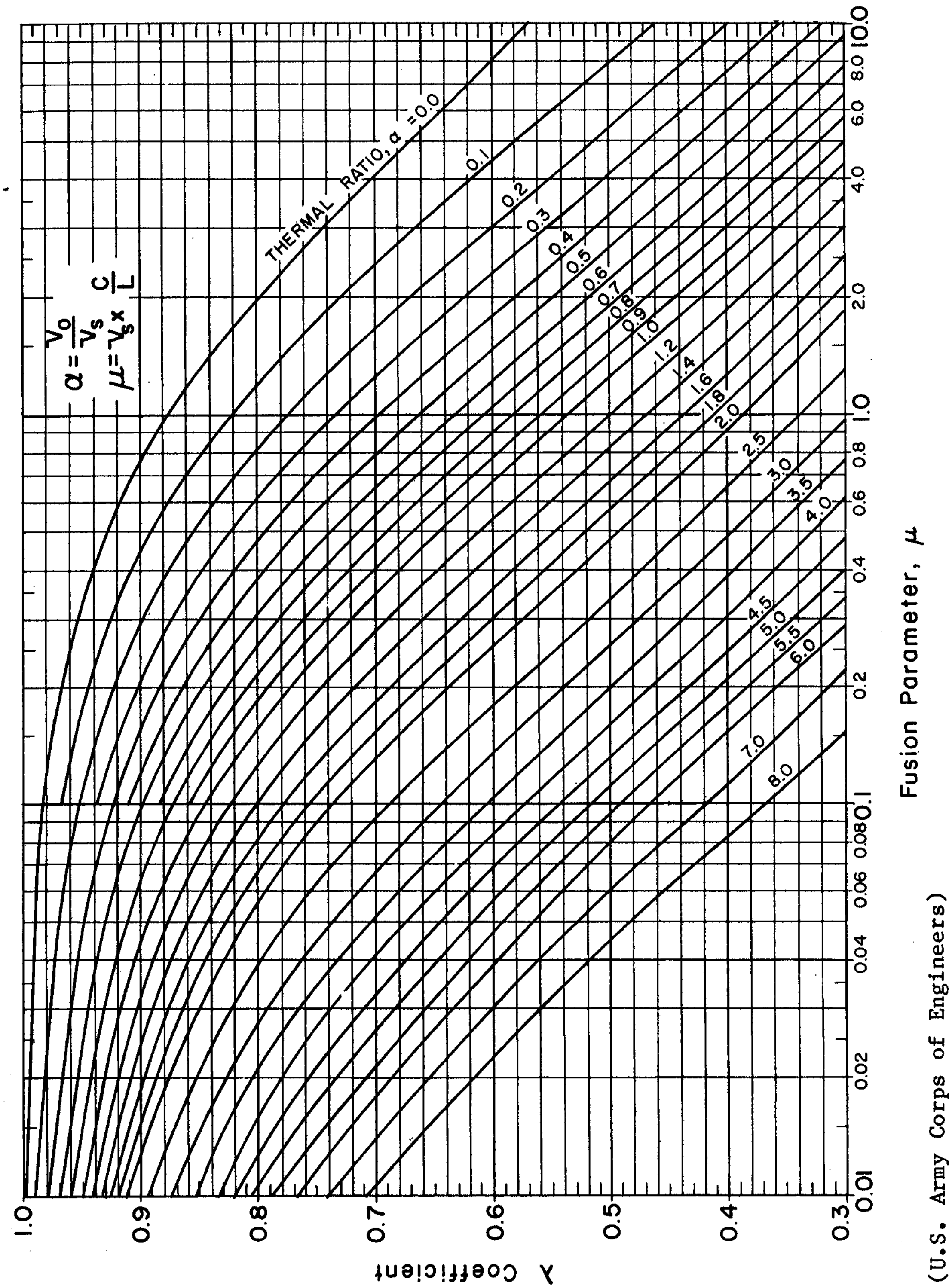


Figure 3-1. λ coefficient in the modified Berggren formula.

remains permanently thawed, or to determine thaw penetration in areas where the ground below a depth of several feet remains permanently frozen. These two conditions are similar in that the temperature gradients are of the same shape, although reversed with respect to the 32°F line. No simple analytical method exists to determine the depth of thaw in seasonal frost areas or the depth of freeze in permafrost areas, and such problems should be referred to HQDA (DAEN-ECE-G) or HQ AFESC. Numerical techniques and computer programs are available to solve more complex problems. Appendix B discusses some thermal computer models for computing freeze and thaw depths. The modified Berggren equation cannot be used successfully to calculate penetration over parts of the season. The modified Berggren equation does not account for any moisture movement that may occur within the soil. This limitation would tend to result in overestimated frost penetration (if frost heave is significant) or underestimated thaw penetration.

(3) *Applicability*. The modified Berggren equation is most often applicable in either of two ways: to calculate the multi-year depth of thaw in permafrost areas or to calculate the depth of seasonal frost penetration in seasonal frost areas. It is also sometimes used to calculate seasonal thaw penetration (active layer thickness) in permafrost areas.

3-3. Homogeneous soils.

The depth of freeze or thaw in one layer of homogeneous soil may be determined by means of the modified Berggren equation. A thin bituminous concrete pavement will not affect the homogeneity of this layer in calculations, but a portland-cement-concrete pavement greater than 6 inches thick should be treated as a multilayered system. In this *example* for homogeneous soils, determine the depth of frost penetration into a homogeneous sandy silt for the following conditions:

—Mean annual temperature (MAT) = 37.2°F.

—Surface freezing index (nF) = 2500 degree-days.

—Length of freezing season (t) = 160 days.

—Soil properties: γ_d = 100 lb/ft^3, w = 15%.

The soil thermal properties are as follows:

—Volumetric latent heat of fusion,

$L = 144(100)(0.15) = 2160\ \text{Btu/ft}^3.$ (eq. 3-2).

—Average volumetric heat capacity,

$C_{avg} = 100[0.17++(0.75 \times 0.15)]$
$= 28.3\ \text{Btu/ft}^3\ °\text{F}.$ (eq 3-3)

—Average thermal conductivity,

K_f = 0.80 Btu/ft hr °F (fig. 2-3)

K_u = 0.72 Btu/ft hr °F (fig. 2-4)

$K_{avg} = 1/2\,(K_u + K_f) = 0.76$ Btu/ft hr °F.

The λ coefficient is as follows:

—Average surface temperature differential,

$v_s = nF/t = 2500/160$
$= 16.6°F$ (16.6°F below 32°F). (eq 3-4)

—Initial temperature differential,

$v_o = MAT - 32 = 37.2 - 32.0 =$
5.2°F (5.2° above 32°F). (eq 3-5)

—Thermal ratio,

$\alpha = v_o/v_s = 5.2/16.6 = 0.33.$ (eq 3-6)

—Fusion parameter,

$\mu = v_s\,(C/L) = 16.6(28.2/2160) = 0.20.$ (eq 3-7)

—Lambda coefficient,

$\lambda = 0.89$ (fig. 3-1). (eq 3-8)

Estimated depth of frost penetration,

$$X = \lambda\sqrt{\frac{48\ K\ nF}{L}} = 0.89\sqrt{\frac{48(0.76)(2500)}{2160}} = 5.8\ \text{ft.} \quad \text{(eq 3-9)}$$

3-4. Multilayer soils.

A multilayer solution to the modified Berggren equation is used for nonhomogeneous soils by determining that portion of the surface freezing (or thawing) index required to penetrate each layer. The sum of the thicknesses of all the frozen (or thawed) layers is the depth of freeze (or thaw). The partial freezing (or thawing) index required to penetrate the top layer is given by

$$F_1\ (\text{or}\ I_1) = \frac{L_1 d_1}{24\lambda_1^2}\left(\frac{R_1}{2}\right) \quad \text{(eq 3-10)}$$

where

d_1 = thickness of first layer (ft)

$R_1 = d_1/K_1$ = thermal resistance of first layer.

The partial freezing (or thawing) index required to penetrate the second layer is

$$F_2 \text{ (or } I_2) = \frac{L_2 d_2}{24\lambda_2^2}\left(R_1 + \frac{R_2}{2}\right). \quad \text{(eq 3-11)}$$

The partial index required to penetrate the n^{th} layer is:

$$F_n \text{ (or } I_n) = \frac{L_n d_n}{24\lambda_n^2}\left(\Sigma R + \frac{R_n}{2}\right) \quad \text{(eq 3-12)}$$

where ΣR is the total thermal resistance above the n^{th} layer and equals

$$R_1 + R_2 + R_3 \ldots + R_{n-1}. \quad \text{(eq 3-13)}$$

The summation of the partial indexes,

$$F_1 + F_2 + F_3 \ldots + F_n \text{(or } I_1 + I_2 + I_3 \ldots + I_n) \quad \text{(eq 3-14)}$$

is equal to the surface freezing index thawing index).

a. In this *example*, determine the depth of thaw penetration beneath a bituminous concrete pavement for the following conditions:

—Mean annual temperature (MAT) = 12°F.

—Air thawing index (I) = 780 degree-days.

—Average wind speed in summer = 7.5 miles per hour (mph).

—Length of thaw season (t) = 105 days.

—Soil boring log:

Since a wind speed of 7-1/2 mph results in an n-factor of 2.0 (fig. 2-10), a surface thawing index nI of 1560 degree-days is used in the computations. The v_s, v_o and α values are determined in the same way as those for the homogeneous case:

$$v_s = 1560/105 = 14.8°F \quad \text{(eq 3-15)}$$

$$v_o = 12.0 - 32.0 = 20.0°F \quad \text{(eq 3-16)}$$

$$\alpha = 20.0/14.8 = 1.35. \quad \text{(eq 3-17)}$$

The thermal properties C, K and L of the respective layers are obtained from figures 2-1 through 2-8.

b. Table 3-1 facilitates solution of the multilayer problem, and in the following discussion, layer 3 is used to illustrate quantitative values. Columns 9, 10, 12 and 13 are self-explanatory. Column 11, $\overline{L}$, represents the average value of L for a layer and is equal to $\Sigma Ld/\Sigma d$ (2581/5.0 = 517). Column 14, $\overline{C}$, represents the average value of C and is obtained from $\Sigma Cd/\Sigma d$ (145/5.0 = 29). Thus $\overline{L}$ and $\overline{C}$ represent weighted values to a depth of thaw penetration given by Σd, which is the sum of all layer thicknesses to that depth.

The fusion parameter μ for each layer is determined from

$$v_s (\overline{C}/\overline{L}) = 14.8 (29/517) = 0.83. \quad \text{(eq 3-18)}$$

The λ coefficient is equal to 0.508 from figure 3-1. Column 18, R_n, is the ratio d/K and for layer 3 equals (3.0/2.0) or 1.5. Column 19, ΣR, represents the sum of the R_n values above the layer under consideration. Column 20, $\Sigma R + (R_n/2)$, equals the sum of the R_n values above the layer plus one-half the R_n value of the layer being considered. For layer 3 this is [1.32 + (1.50/2)] = 2.07. Column 21, nI, represents the number of degree-days required to thaw the layer being considered and is determined from

Layer	Depth (ft)	Material*	Dry unit weight (lb/ft³)	Water content (%)
1	0.0-0.4	Asphaltic concrete	138	--
2	0.4-2.0	GW-GP	156	2.1
3	2.0-5.0	GW-GP	151	2.8
4	5.0-6.0	SM	130	6.5
5	6.0-8.0	SM-SC	122	4.6
6	8.0-9.0	SM	116	5.2

*In accordance with Unified Soil Classification System.

Table 3-1. Multilayer solution of modified Berggren equation.

Table 3-1. Multilayer solution of modified Berggren equation (U.S. Army Corps of Engineers).

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)	(21)	(22)
Layer	γ_d	w	d	Σd	C	K	L	Ld	ΣLd	$\bar{L}$	Cd	ΣCd	$\bar{C}$	μ	λ	λ^2	R_n	ΣR	$\Sigma R + \frac{R_n}{2}$	nI	ΣnI
1	138	--	0.4	0.4	28	0.86	0	0	0	--	12	12	--	--	--	--	0.46	0	0.23	--	--
2	156	2.1	1.6	2.0	29	1.85	470	751	751	376	46	58	29	1.15	0.455	0.207	0.86	0.46	0.89	134	134
3	151	2.8	3.0	5.0	29	2.00	610	1830	2581	517	87	145	29	0.83	0.508	0.258	1.50	1.32	2.07	612	746
4	130	6.5	1.0	6.0	28	1.65	1220	1220	3801	633	28	173	29	0.68	0.537	0.288	0.60	2.82	3.12	551	1297
5a	122	4.6	1.0	7.0	25	0.64	808	808	4609	658	25	198	28	0.63	0.552	0.305	1.56	3.42	4.20	465	1762
5b	122	4.6	0.6	6.6	25	0.64	808	485	4286	650	15	188	28	0.64	0.550	0.303	0.94	3.42	3.89	260	1557

$\alpha = 1.35$ $v_s = 14.8°F$ nI = 1560 degree-days

$$I_2 = \frac{(470)(1.6)}{(24)(0.207)}(0.89) = 134$$

$$I_3 = \frac{(610)(3.0)}{(24)(0.258)}(2.07) = 612$$

$$I_4 = \frac{(1220)(1.0)}{(24)(0.288)}(3.12) = 551$$

$$I_{5a} = \frac{(808)(1.0)}{(24)(0.305)}(4.20) = 465$$

$$I_{5b} = \frac{(808)(0.6)}{(24)(0.303)}(3.89) = 260$$

Total thaw penetration = 6.6 feet

$$nI = \frac{Ld}{24\lambda^2}\left(\Sigma R + \frac{R_n}{2}\right). \qquad \text{(eq 3-19)}$$

For layer 3,

$$nI_3 = \frac{(610)(3.0)}{24(0.508)^2}(2.07) \qquad \text{(eq 3-20)}$$

$$= 612 \text{ degree-days}$$

The summation of the number of degree-days required to thaw layers 1 through 4 is 1297, leaving (1560 - 1297 =) 263 degree-days to thaw a portion of layer 5. A trial-and-error method is used to determine the thickness of the thawed part of layer 5. First, it is assumed that 1.0 feet of layer 5 is thawed (designated as layer 5a). Calculations indicate 465 degree-days are needed to thaw 1.0 foot of layer 5 or (465 - 263 =) 202 degree-days more than available. A new layer, 5b, is then selected by the following proportion

$$(263/465)1.0 = 0.57 \text{ ft (try 0.6 ft)}. \qquad \text{(eq 3-21)}$$

This new thickness results in 260 degree-days required to thaw layer 5b or 3 degree-days less than available. Further trial-and-error is unwarranted and the total estimated thaw penetration would be 6.6 feet. A similar technique is used to estimate frost penetration in a multilayer soil profile.

3-5. Effect of snow and vegetative cover.

Thermal properties of snow and vegetative covers are extremely variable in both time and space. Both materials tend to act as insulators and retard heat transfer at the air-ground interface. In freeze-thaw computations, snow and vegetative surface materials are treated as separate layers in the multilayer solution of the modified Berggren equation, with snow cover thickness estimated seasonally. The tabulation below presents average thermal properties of snow applicable for calculation in the noted regions if a better data base is not available. In the absence of site-specific data, figures 2-5 and 2-6 should be used to estimate the thermal conductivities of vegetative surface cover.

3-6. Surface temperature variations.

The temperatures at the air-ground interface are subject to daily and seasonal fluctuations. Precipitation, insolation, air temperature and turbulence contribute to these variations in surface temperature. To facilitate mathematical calculations, two assumptions are commonly made regarding the temperatures at the upper boundary: 1) a sudden step change occurs in surface temperature or 2) the surface temperature change is sinusoidal. The sinusoidal variation of temperature over a year closely approximates actual conditions; however, it is amenable to hand calculations only if latent heat effects are negligible. Solutions and examples for both conditions are given below.

a. Sudden step change. This involves a sudden change in the surface temperature of a mass that was initially at a constant, uniform temperature. The sudden step change was used to establish the boundary conditions for heat flow in the modified Berggren equation given in paragraph 3-2. If the influence of latent heat is not involved, or is assumed negligible, the following equation may be used:

$$T_{(x,t)} = T_s + (T_o - T_s)\,\text{erf}\left(\frac{x}{2\sqrt{at}}\right) \qquad \text{(eq 3-22)}$$

Region	*Unit weight (lb/ft³)*	*K (Btu/ft) hr °F)*	*C (Btu/ft³ °F)*	*L (Btu/ft³)*
Interior Alaska	16	0.11	8	2300
Canadian Archipelago, N. Alaskan coast, and temperate regions	20	0.18	10	2880
Northern Greenland	22	0.20	11	3170

where

$T_{(x,t)}$ = temperature at depth x, at time (°F)

T_s = suddenly applied constant surface temperature (°F)

T_o = initial uniform temperature of the mass (°F)

erf = mathematical expression, termed the error function, which is frequently used in heat flow computations (dimensionless)

x = depth below surface (ft)

a = thermal diffusivity of the mass (ft^2/day = K/C)

t = time after application of sudden change in surface temperature (days).

Figure 3-2 gives the relationship between (x/2 $\sqrt{at}$) and erf (x/2 $\sqrt{at}$). The expression for the error function is shown in appendix A and figure 3-2.

In this *example* of a sudden step change, a highly frost-susceptible subgrade is covered with a 2-foot thick, non-frost-susceptible gravel pad. Both soils are at an initial temperature of 20°F. If the

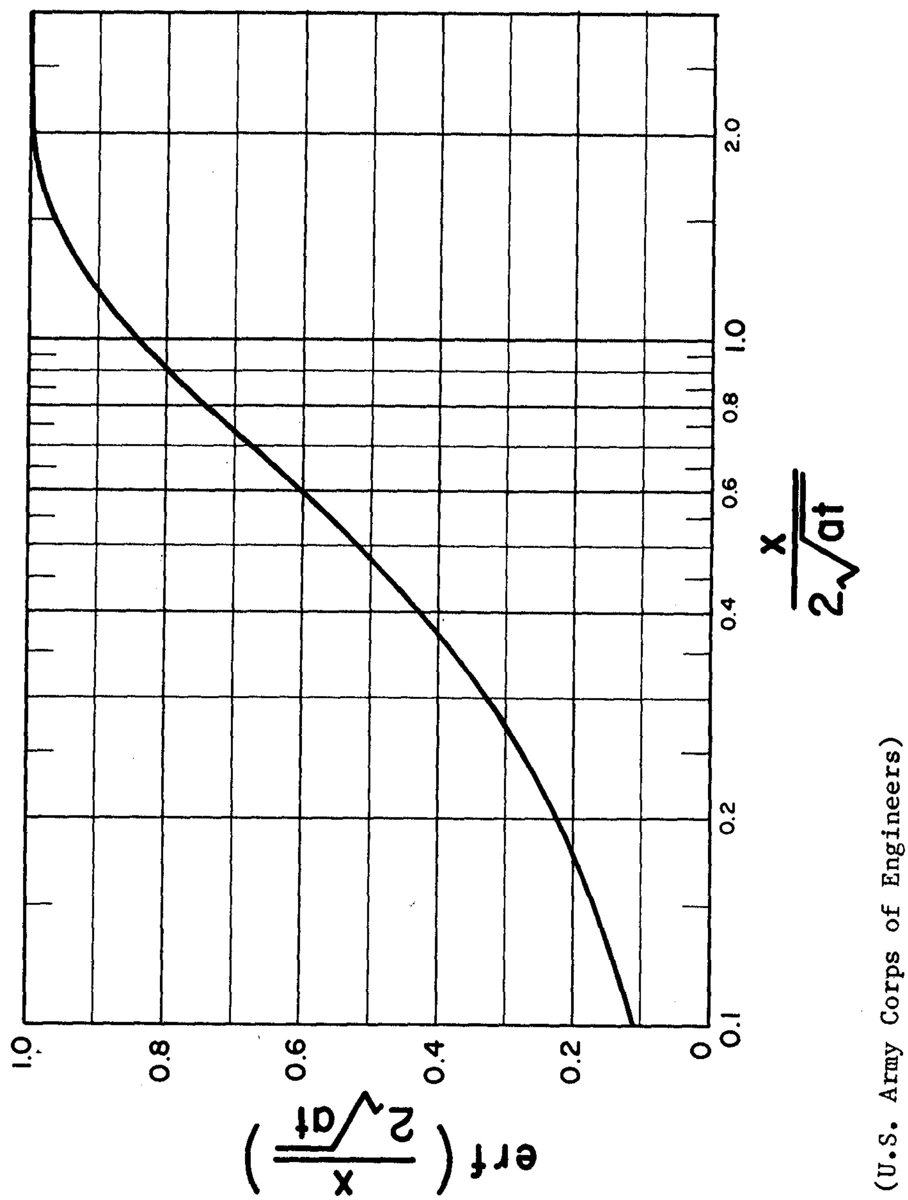


Figure 3-2. Relationship between (x/2 $\sqrt{at}$) and erf (x/2 $\sqrt{at}$).

surface of the gravel is suddenly heated to and maintained at 70°F for a number of days, estimate the temperature at the gravel-subgrade interface after one day. The gravel material is very dry and latent heat may be ignored. The thermal conductivity of the gravel is 1.0 Btu/ft hr °F and the volumetric heat capacity is 25 Btu/ft³. The thermal diffusivity of the gravel is (K/C = 1.0/25) == 0.04 ft²/hr., or 0.96 ft²/day, and $x/2\sqrt{at} = (2.0/2\sqrt{0.96 \times 1}) = 1.02$. From figure 3-2, erf $(x/2\sqrt{at})$ is equal to 0.85, and the interface temperature T is [70 + (20 - 70)0.85] = 27.5°F.

b. Sinusoidal change. A surface temperature variation that is nearly sinusoidal repeats itself periodically for a surface exposed to the atmosphere. For most problems in this manual, the sinusoidal variation of concern occurs over an annual cycle. If latent heat is not involved or is assumed negligible, the following equation may be used:

$$A_x = A_o \exp\left(-x\sqrt{\frac{\pi}{aP}}\right) \qquad \text{(eq 3-23)}$$

where

A_x = amplitude of temperature wave at depth x (°F).

A_o = amplitude of the surface temperature wave above or below the average annual temperature (°F)

x = depth below surface (ft)

a = thermal diffusivity of the mass (ft²/day)

P = period of sine wave (365 days).

The sinusoidal temperature pattern is assumed to exist at all levels to a depth where there is no temperature change. The temperature waves lag behind the surface wave, and the amplitude of the sinusoidal waves decreases with depth below the surface. The phase lag is determined by $t_x = (x/2)(\sqrt{365/\pi a})$. Typical temperature-time curves for a surface and at a depth x are shown in figure 3-3.

In the following *example* of a sinusoidal temperature change, the surface temperature of an 8-foot-thick concrete slab varies from 60° to -40°F during the year. Determine the maximum temperature at the base of the slab assuming a diffusivity of 1.0 ft²/day for the concrete. The average annual temperature is [60 + (-40)]/2 = 10°F and the surface amplitude is (60 - 10) = 50°F. The amplitude at an 8-foot depth equals

$$A_x = 50 \exp\left[-8\sqrt{\frac{\pi}{(1.0)(365)}}\right] = 50\,e^{-0.742} = 24°F. \qquad \text{(eq 3-24)}$$

The maximum temperature at 8 feet is (10 + 24) = 34°F. The time lag t_x between the maximum temperature at the surface and 8 feet is

$$t_x = \frac{8}{2}\sqrt{\frac{365}{\pi(1.0)}} = 43 \text{ days (about 6 weeks).} \qquad \text{(eq 3-25)}$$

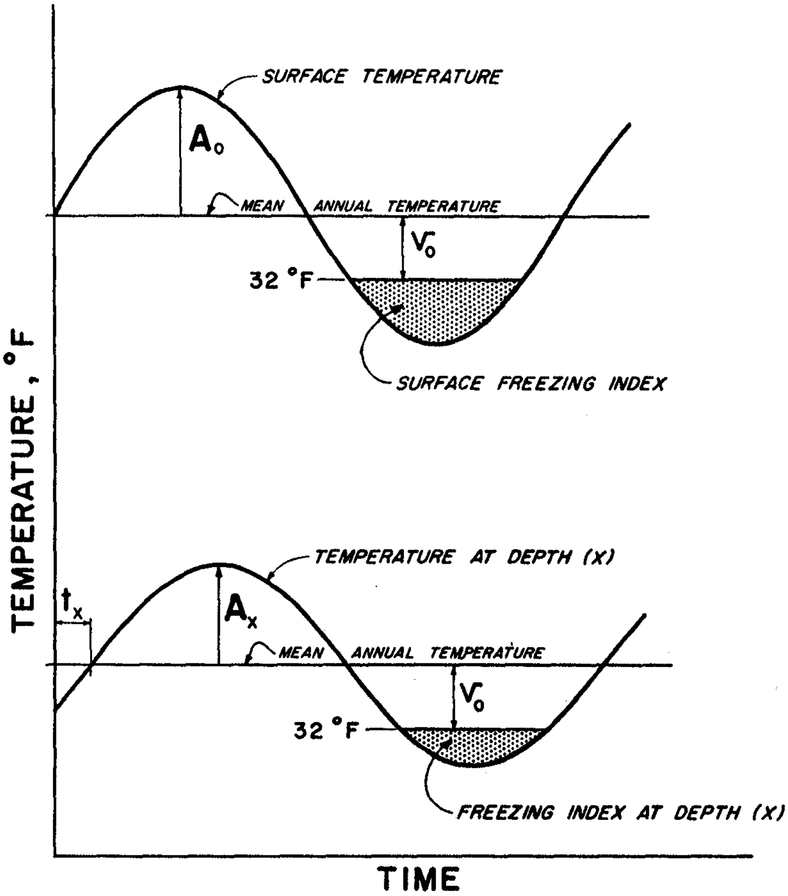
(Note: latent heat would increase the time lag and decrease the amplitude.)

c. Nonuniform layers. The method of equivalent thickness is used to find the temperature at a point below a number of layers of different thermal properties. This technique assumes a negligible effect of latent heat, and involves reduction of each layer to an equivalent material thickness by setting the ratio of the thicknesses equal to the ratio of square roots of the thermal diffusivities. For *example*, determine the equivalent gravel thickness for the three layers shown, assuming all materials are unfrozen.

The following table shows that 4.75 feet of the nonuniform materials can be considered equivalent to 5.4 feet of gravel for heat-flow purposes. This equivalent thickness and the thermal diffusivity of the gravel are used to calculate temperatures at the base of the gravel layer by either the step-change or sinusoidal method.

3-7. Converting indexes into equivalent sine wave of temperature.

Some problems may require the use of the sinusoidal temperature variation technique, given only the freezing or thawing indexes and the average annual temperature. These indexes may be converted into a sine curve of temperature to give the same index values and the same mean temperature. For example, convert the monthly average temperature data for Fairbanks, Alaska, shown in figure 2-9 into an equivalent sine wave. The relationship between the sinusoidal amplitude, freezing index, thawing index and aver-



(U.S. Army Corps of Engineers)

Figure 3-3. Sinusoidal temperature pattern.

Material	*Dry Unit weight,* γ_d (lb/ft^3)	*Water content, w (%)*	*Thermal conductivity, K (Btu/ft hr° F)*	*Volumetric heat capacity, C (Btu/ft^3 ° F)*	*Thermal diffusivity,* a = K/C (ft^2/hr)
Concrete	--	--	1.0	33.0	0.033
Sand	120	2	0.8*	23 †	0.035
Gravel	135	4	1.5*	28 †x	0.054

*From figure 2-2.
†$C = \gamma_d\,(0.17 + w/100)$.

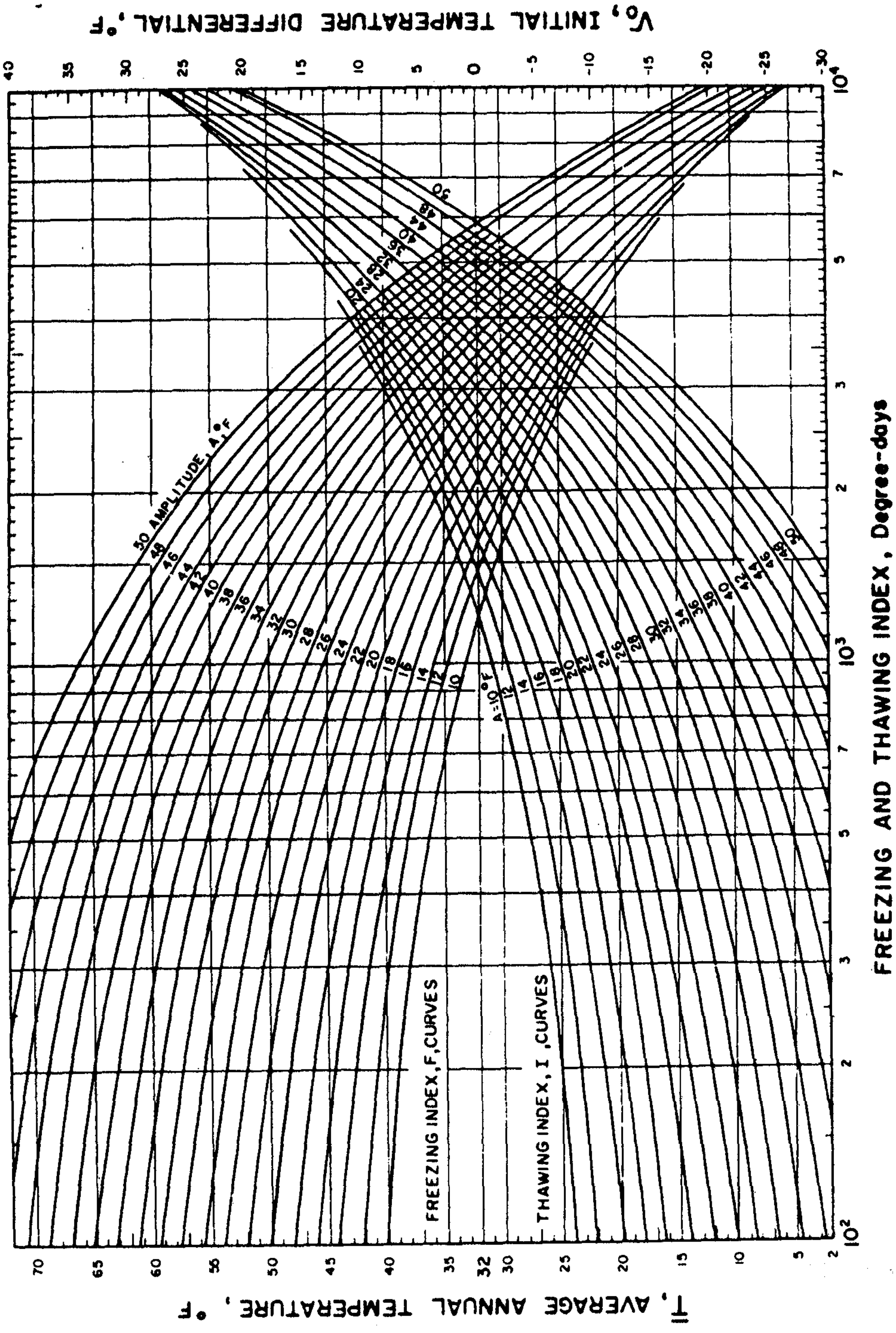
Material	*Thickness (ft)*	*Thermal diffusivity (ft^2/hr)*	$\frac{\sqrt{a_g}}{\sqrt{a_m}}$*	*Equivalent gravel thickness (ft)*
Concrete	1.75	0.033	1.3	2.3 (1.3 × 1.75)
Sand	0.50	0.035	1.2	0.6
Gravel	2.50	0.054	1.00	2.50
Total thickness	4.75			5.4

*The subscript g refers to the gravel layer and the subscript m refers to the other material layer.

age annual temperature is shown in figure 3-4.

The average temperature at Fairbanks for October 1949 to September 1950 was 27°F. By use of the freezing index of 5240 degree-days, the sinusoidal amplitude is found to be 37.0°F. The equation of the sine wave is

$$T = 27.0 - 37.0 \sin 2\pi f t \qquad \text{(eq 3-26)}$$
$$= 27.0 - 37.0 \sin 0.0172\, t \text{ (radians)}$$



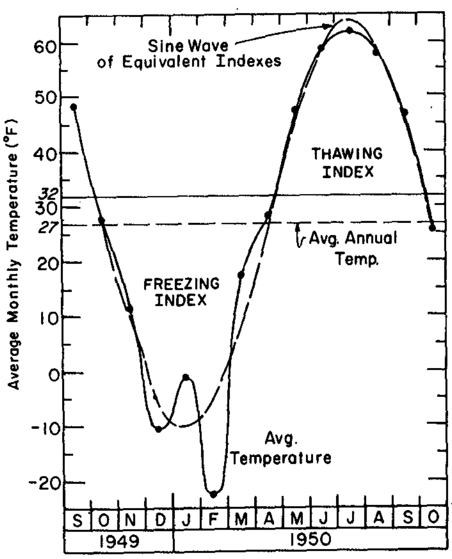
(U.S. Army Corps of Engineers)

Figure 3-4. Indexes and equivalent sinusoidal temperature.

where

f = frequency, 1/365 cycles per day
t = time from origin in days. (Origin of curve is located at a point where T intersects the average annual temperature on its way downward toward the yearly minimum.)

If the thawing index of 3240 degree-days had been used, the sinusoidal temperature amplitude would be 35.5°F. The actual temperature curve for Fairbanks, Alaska, and the equivalent sine wave computed from the freezing index are plotted in figure 3-5. This illustration makes use of air indexes but a sine wave could be determined for surface indexes by multiplying the air indexes with appropriate n-factors. Note that the mean annual ground surface temperature may be substantially different (frequently higher) from the mean annual air temperature because the freezing n-factor is generally not equal to the thawing n-factor. If the long-term mean monthly temperature had been used instead of the average monthly temperatures for the 1949-1950 period, the correlation between the actual temperature curve and the equivalent sine curve would practically coincide, as shown in figure 3-6.



(U.S. Army Corps of Engineers)

Figure 3-5. Average monthly temperatures for 1949-1950 and equivalent sine wave, Fairbanks, Alaska.

3-8. Penetration of freeze or thaw beneath buildings.

The penetration of freeze or thaw beneath buildings depends largely on the presence or absence of an airspace between the building floor and the ground as discussed below and in TM 5-852-4/AFM 88-19, Chapter 4.

a. Building floor placed on ground. When the floor of a heated building is placed directly on frozen ground, the depth of thaw is determined by the same method as that used to solve a multilayer problem when the surface is exposed to the atmosphere, except that the thawing index is replaced by the product of the time and the differential between the building floor temperature and 32°F. For *example*, estimate the depth of thaw after 1 year for a building floor consisting of 8 inches of concrete, 4 inches of insulation and 6 inches of concrete, placed directly on a 5-foot-thick sand pad overlying permanently frozen silt for the following conditions:

—Mean annual temperature = 20°F.
—Building floor temperature = 65°F.
—Sand pad: γ_d = 133 lb/ft^3, w = 5%.
—Frozen silt: γ_d = 75 lb/ft^3, w = 45%.
—Concrete: K = 1.0 Btu/ft hr °F, C = 30 Btu/ft^3 °F.
—Insulation: K = 0.033 Btu/ft hr °F, C = 1.5 Btu/ft^3 °F.

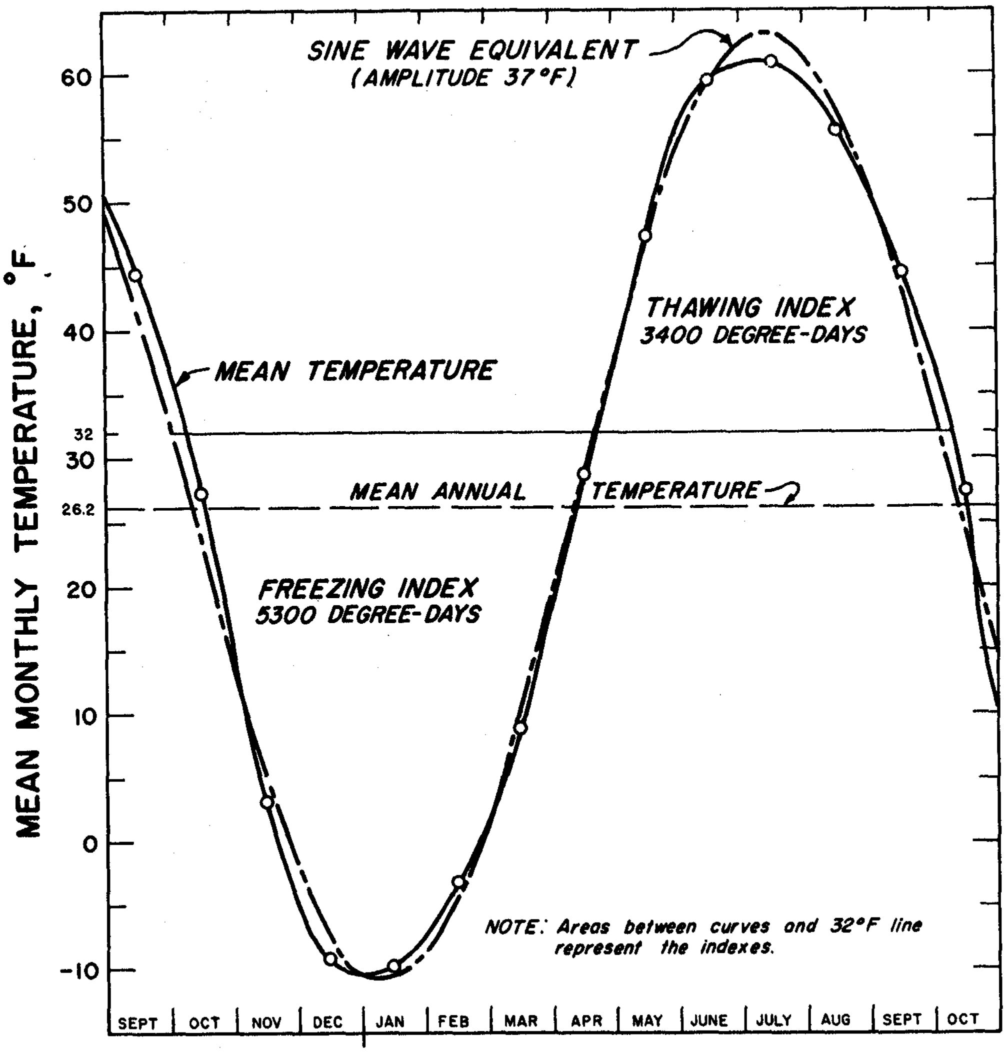
The resistances of the three floor layers are in series, and the floor resistance R_f is the sum of the three layer resistances:

$$R_f = \frac{d}{k} + \frac{8}{(12)(1.0)} + \frac{4}{(12)(0.033)} + \frac{6}{(12)(1.0)} = 11.2\ \text{ft}^2\ \text{ft}° \qquad \text{(eq 3-27)}$$

The average volumetric heat capacity of the floor system is

$$C_f = \frac{(30)(8) + (1.5)(4) + (30)(6)}{8 + 4 + 6} = 23.7\ \text{Btu/ft}^3\ °\text{F} \qquad \text{(eq 3-28)}$$

The solution to this problem, shown in table 3-2, predicts a total thaw depth of 7.8 feet. This solution did not consider edge effects, i.e., a long narrow building will have lesser depth of thaw than a square building with the same floor because of the difference in lateral heat flow.



(U.S. Army Corps of Engineers)

Figure 3-6. Long-term mean monthly temperatures and equivalent sine wave, Fairbanks, Alaska.

b. Airspace below building

(1) An unskirted airspace between the heated floor of building and the ground will help prevent degradation of underlying permafrost. The airspace insulates the building floor from the ground and acts as a convective passage for flow of cold air that dissipates heat from the floor system and the ground. The depth of thaw is calculated by means of the modified Berggren equation for either a homogeneous or multilayered soil system, as applicable. An n-factor of 1.0 is recommended to determine the surface thawing index beneath the shaded area of an elevated building.

(2) There is no simple mathematical expression for analyzing the heat flow in a ventilated floor system that has ducts or pipes installed within the floor or at some depth beneath the floor, with air circulation induced by stack effect. The depth to which freezing temperatures will penetrate is computed with the modified Berggren equation, except that the air freezing index at the outlet governs. This index is influenced by a number of design variables, i.e., average daily air temperatures, inside building temperatures, floor and duct or pipe system design, temperature and velocity of air in the system, and stack height. Cold air

Table 3-2. Thaw penetration beneath a slab-on-grade building constructed on permafrost.

Table 3-2. Thaw penetration beneath a slab-on-grade building constructed on permafrost (U.S. Army Corps of Engineers).

Layer	γ_d	w	d	Σd	C	K	L	Ld	ΣLd	$\bar{L}$	Cd	ΣCd	$\bar{C}$	μ	λ	λ^2	R_n	ΣR	$\Sigma R + \frac{R_n}{2}$	nI	ΣnI
Floor	--	--	1.5	1.5	24	--	0	0	0	0	36	--	--	--	--	--	11.20	0	0	--	--
Sand	133	5.0	5.0	6.5	28	1.54	960	4800	4800	738	140	176	27	1.21	0.68	0.463	3.25	11.20	12.82	5540	5540
Silt a	72	45.0	1.5	8.0	37	0.90	4650	6970	11770	1470	55	231	29	0.65	0.77	0.593	1.67	14.45	15.29	7480	13020
Silt b	72	45.0	1.3	7.8	37	0.90	4650	6050	10850	1390	48	224	29	0.69	0.765	0.586	1.44	14.45	15.17	6520	12060

$v_o = 32 - 20 = 12°F$ $\quad$ $v_s = 65-32 = 33°F$ $\quad$ $\alpha = 12/33 = 0.36$

Surface thawing index (nI) = 33 x 365 = 12050 degree-days

$$nI\,(Sand) = \frac{4800}{24(0.463)}(12.82) = 5540 \text{ degree-days}$$

$$nI\,(Silt\ a) = \frac{6970}{24(0.593)}(15.29) = 7480 \text{ degree-days}$$

$$nI\,(Silt\ b) = \frac{6050}{24(0.586)}(15.17) = 6520 \text{ degree-days}$$

Total thaw penetration = 7.8 feet

passing through the ducts acquires heat from the duct walls and experiences a temperature rise as it moves through the duct, and the air freezing index is reduced at the outlet. Field observations indicate that the inlet air freezing index closely approximates the site air freezing index. The freezing index at the outlet must be sufficient to counteract the thawing index and ensure freeze-back of foundation soils.

(3) As an *example*, determine the required thickness of a gravel pad beneath the floor section shown in figure 3-7 to contain all thaw penetration. Also determine the required stack height to ensure freeze-back of the pad on the outlet side of the ducts. The conditions for this example follow:

- Duct length, l = 220 ft.
- Gravel pad: $\gamma_d = 125$ lb/ft^3, w = 2.5%.
- Outlet mean annual temperature = 32°F 0, w = 2.5%. (conservative assumption).
- Minimum site freezing index = 4000 degree-days.
- Freezing season = 215 days.
- Thawing season = 150 days (period during which ducts are closed).
- Building floor temperature = 60°F.
- Thermal conductivity of concrete, K_c = 1.0 Btu/ft hr °F.
- Thermal conductivity of insulation, K_i = 0.033 Btu/ft hr °°°°F.

(a) The required thickness is determined by the following equation, derived from the modified Berggren equation:

$$X = KR_f \left[\sqrt{1 + \frac{48 r^2 I_f}{KL(R_f)^2}} - 1 \right] \qquad \text{(eq 3-29)}$$

Figure 3-7. Schematic of ducted foundation.

where

K = average thermal conductivity of gravel
= 1/2(0.7 + 1.0) = 0.85 Btu/ft hr °F

R_T = thermal resistance of floor system

$$= \frac{18}{12(1.0)} + \frac{4}{12(0.033)} + \frac{12}{12(1.0)}$$

$$= 12.5 \text{ ft}^2 \text{ hr °F/Btu} \qquad \text{(eq 3-30)}$$

(In the computations the dead airspace is assumed equivalent to the thermal resistance of concrete of the same thickness.)

λ = factor in modified Berggren equation = 0.97 (conservative assumption)

I_f = thawing index at floor surface = (60 - 32)(150) = 4200 degree-days

L = latent heat of gravel = 144(125)(0.025) = 450 Btu/ft³

then

$$X = (0.85)(12.5)\left[\sqrt{1 + \frac{(48)(0.97)^2(4200)}{(0.85)(450)(12.5)^2}} - 1\right]$$

$$= 11.0 \text{ ft.} \qquad \text{(eq 3-31)}$$

(b) Thus the total amount of heat to be removed from the gravel pad by cold-air ventilation during the freezing season with ducts open is equal to the latent and sensible heat contained in the thawed pad. The heat content per square foot of pad is determined as follows:

— Latent heat, (X)(L) = (11.0)(450) = 4950 Btu/ft²

— Sensible heat (10 percent of latent heat, based upon experience) = 495

— Total heat content:
5445 Btu/ft².

The ducts will be open during the freezing season (215 days), and the average rate of heat flow from the gravel during this season is equal to 5445/215 × 24 = 1.0 Btu/ft² hr. The average thawing index at the surface of the pad is

$$\frac{L X^2}{48\lambda^2 K} = \frac{(450)(11.0)^2}{48(0.97)^2(0.85)} = 1420 \text{ degree-days.} \qquad \text{(eq 3-32)}$$

This thawing index must be compensated for by an equal freezing index at the duct outlet on the surface of the pad to assure freeze-back. The average pad surface temperature at the outlet end equals the ratio

$$\frac{\text{Required freezing index}}{\text{Length of freezing season}}$$

$$= \frac{1420}{215}$$

= 6.6°F below 32°F or 25.4°F.

The inlet air during the freezing season has an average temperature of

$$\frac{\text{Air freezing index}}{\text{Length of freezing season}} = \frac{4000}{215}$$

= 18.6°F below 32°F or 13.4°F.

Therefore, the average permissible temperature rise T_R along the duct is (25.4 - 13.4) = 12.0°F.

(c) The heat flowing from the floor surface to the duct air during the winter is equal to the temperature difference between the floor and duct air divided by the thermal resistance between them. The thermal resistance R is calculated as follows:

$$R = \frac{X_c}{K_c} + \frac{X_i}{K_i} + \frac{1}{h_{rc}} = \frac{14}{(12)(1.0)}$$

$$+ \frac{4}{(12)(0.033)} + \frac{1}{1.0} = 12.3 \text{ hr ft}^2 \text{ °F/Btu} \qquad \text{(eq 3-33)}$$

where

X_c = thickness of concrete (ft)

X_i = thickness of insulation (ft)

h_{rc} = surface transfer coefficient between duct wall and duct air

(For practical design, h_{rc} = 1.0 Btu/ft² hr °F and represents the combined effect of convection and radiation. At much higher air velocities, this value will be slightly larger; however, using a value of 1.0 will lead to conservative designs). The average heat flow between the floor and inlet duct air is [(60 - 13.4)/12.3] = 3.8 Btu/ft² hr, and between the floor and outlet duct air is [(60 - 25.4)/12.3] = 2.8 Btu/ft² hr. Thus the average rate of heat flow from the gravel pad to the duct air is 1.0 Btu/ft² hr. The total heat flow ϕ to the duct air from the floor and gravel pad is (3.3 + 1.0) = 4.3 Btu/ft² hr. The heat flow to the duct air must equal the heat removed by the duct air.

Heat added = heat removed

$$\phi \ell m = 60V A_d \rho c_p T_R. \qquad \text{(eq 3-34)}$$

Thus the average duct air velocity required to extract this quantity of heat (4.3 Btu/ft^2 hr) is determined by the equation:

$$V = \frac{\phi \ell m}{60\, A_d\, \rho\, C_p T_R} \text{ ft/minute} \qquad \text{(eq 3-35)}$$

where

- ϕ = total heat flow to duct air (4.3 Btu/ft^2 hr)
- ℓ = length of duct (220 ft)
- m = duct spacing (2.66 ft)
- A_d = cross-sectional area of duct (1.58 ft^2)
- ϕ = density of air (0.083 lb/ft^3 [figure 3-10])
- c_p = specific heat of air at constant pressure (0.24 Btu/lb °F)
- T_R = temperature rise in duct air (12°F).

Substitution of appropriate values gives a required air velocity

$$V = \frac{(4.3)(220)(2.66)}{(60)(1.58)(0.083)(0.24)(12.0)} = 111 \text{ ft/minute.} \qquad \text{(eq 3-36)}$$

(d) The required air flow is obtained by a stack or chimney effect, which is related to the stack height. The stack height is determined by the equation

$$h_d = h_v + h_f \qquad \text{(eq 3-37)}$$

where

$$h_d = \frac{\rho\, \varepsilon\, H(T_c - T_o)}{5.2(T_c + 460)} \text{ inches of water}$$

(natural draft head)

- ρ = density of air at average duct temperature (lb/ft^3)
- ε = efficiency of stack system (%). This factor provides for

Figure 3-8. Properties of dry air at atmospheric pressure.

friction losses within the chimney

H = stack height (ft)

T_c = temperature of air in stack (°F)

T_o = temperature of air surrounding stack (°F)

$h_v = \left(\frac{V}{4000}\right)^2$ inches of water (velocity head)

V = velocity of duct air (ft/minute)

$h_f = f' \frac{\ell_e}{D_e} h_v$ inches of water (friction head)

f' = friction factor (dimensionless)

ℓ_e = equivalent duct length (ft)

D_e = equivalent duct diameter (ft).

The technique used to calculate the friction head is

$$D_e = \frac{4(\text{cross-sectional area of duct in ft}^2)}{\text{perimeter of duct in ft}} = \frac{4(1.58)}{\frac{2}{12}\left(\frac{18+20}{2}+12\right)}$$

$$= 1.22 \text{ ft.} \qquad \text{(eq 3-38)}$$

The equivalent length of the duct is equal to the actual length ℓ_s plus an allowance ℓ_b for bends and entry and exit. Each right-angle bend has the effect of adding approximately 65 diameters to the length of the duct, and entry and exit effects add about 10 diameters for each entry or exit. In this example the total allowance ℓ_b for these effects is [2(65 + 10) =] 150 diameters, which is added to the length of the straight duct. The estimated length of straight duct ℓ_s is

5 ft	(assumed inlet open length)
220 ft	(length of duct beneath floor)
15 ft	(assumed stack height)
240 ft	

$$\ell_e = \ell_s + \ell_b \qquad \text{(eq 3-39)}$$

$$\ell_e = 240 + (150 \times 1.22) = 423 \text{ ft.}$$

The friction factor f' is a function of Reynolds number N_R and the ratio e/D_e. A reasonable absolute roughness factor e of the concrete duct surface is 0.001 feet, based on field observations. Suggested values of e for other types of surfaces are given in the *ASHRAE Data and Guide Book*. The effect of minor variations in e on the friction factor is small, as noted by examining the equation below used to calculate the friction factor f'. Reynolds number is obtained from the equation

$$N_R = \frac{V(a' + 0.25\,D_e)}{\nu} \qquad \text{(eq 3-40)}$$

where

$$N_R = \frac{(111 \times 60)(1.0 + 0.25 \times 1.22)}{0.49} = 17{,}700$$

V = average duct velocity (ft/hr)

a' = shortest dimension (ft)

ν = kinematic viscosity (ft^2/hr at 19.4°F [fig. 3-8]).

The friction factor f' is obtained by solving the equation

$$f' = 0.0055[1 + (20{,}000 \times \frac{e}{D_e} + \frac{10^6}{N_R})^{1/3}]$$

$$= 0.0055[1 + (20{,}000 \times \frac{0.001}{1.22} + \frac{10^6}{17{,}700})^{1/3}]$$

$$= 0.0285. \qquad \text{(eq 3-41)}$$

Therefore, the friction head is

$$h_f = f' \times \frac{\ell_e}{D_e} \times h_v \qquad \text{(eq 3-42)}$$

$$= 0.0285 \times \frac{423}{1.22} \times h_v = 9.8\,h_v.$$

The draft head required to provide the desired velocity head and to overcome the friction head is furnished by the chimney or stack effect. The draft head h_d is obtained as follows:

$$h_d = h_v + h_f = h_v + 9.8\,h_v \qquad \text{(eq 3-43)}$$

$$= 10.8\,h_v$$

$$= 10.8\left(\frac{V}{4000}\right)^2$$

$$= 10.8\left(\frac{111}{4000}\right)^2 = 8.31\times10^{-3} \text{ inches of water.}$$

The stack height required to produce this draft head is

$$H = \frac{5.2\,h_d(T_c + 460)}{\rho\,\varepsilon\,(T_c - T_o)}$$

$$= \frac{(5.2)(8.31\times10^{-3})(25.4 + 460)}{(0.083)(0.80)(25.4 - 13.4)}$$

$$= 26 \text{ ft} \qquad \text{(eq 3-44)}$$

where

ρ = 0.083 lb/ft^3

T_c = 25.4°F

T_o = 13.4°F

ε = 80% (found to be a reasonable design value based on observations over an entire season)

h_d = 8.31×10^{-3} inches of water.

(e) If the stack is too high for the structure, a greater thickness of insulation could be used. In this example, the effect of increasing the insulation thickness by one-half would result in lowering the stack height by five-eighths.

(f) This first approximated stack height is next incorporated in the calculation of the length of straight duct ℓ_s, and the newly obtained ℓ_e is used to recalculate the friction head h_f. By trial-and-error, the final calculated stack height is found to be 26.5 ft.

(g) The stack height is an important variable because an increase in stack height will increase the duct airflow. Circulation of air through the ducts results from 1) a density difference between the air inside the duct and that outside the building, 2) a pressure reduction at the outlet end attributable to the stack effect, 3) a positive pressure head at the inlet end when wind blows directly into the intake stack opening, and 4) a negative pressure head at the outlet when wind passes over the exhaust stack opening. Draft caused by wind is highly erratic and unpredictable and should not be considered in design; however, the vents should be cowled to take advantage of any available velocity head provided by the wind. If sufficient air cannot be drawn through ducts by natural draft, mechanical blowers could be specified or consideration given to alternating airflow in the ducts.

3-9. Use of thermal insulating materials.

An insulating layer may be used in conjunction with a non-frost-susceptible material to reduce the thickness of fill required to keep freezing or thawing temperatures from penetrating into an underlying frost-susceptible soil. As in the example of paragraph 3-8*a*, the thermal resistance of the pavement and insulation layers are added to obtain total resistance, and the latent heat effect of a combined pavement and insulation layer is assumed negligible. (TM 5-818-2/AFM 88-6, Chap. 4 discusses in detail the design of insulated pavements.) If the insulating material will absorb water, its insulating effectiveness will be reduced considerably (as discussed in TM 5-852-4/AFM 88-19, Chap. 4). Limited field tests indicate that the heat-flow resistance of a portland-cement-concrete pavement overlying a high-quality insulating layer is more complicated than simple addition of resistances, but until sufficient data are obtained for validation, treatment of resistances in series is recommended.

a. Example. A pavement consists of 14 inches of portland-cement concrete placed on a 6-foot gravel base course. Frost penetrated 3.2 feet into the underlying silt subgrade. Determine the thickness of insulation required to prevent frost penetration into the subgrade for the following conditions.

- Mean annual temperature = 35.3°F.
- Air freezing index = 3670 degree-days
- Freezing season = 170 days
- Concrete: K = 1.0 Btu/ft hr °F, C = 0.30 Btu/ft^3 °F
- Insulation: K = 0.024 Btu/ft hr °F, C = 0.28 Btu/ft^3 °F
- Gravel base: γ_d = 130 lb/ft^3, w = 4%
- Silt subgrade: γ_d = 100 lb/ft^3, w = 10%.

Surface freezing index = 0.75 × 3670 = 2752 degree-days. From the known data

$$v_s = \frac{2752}{170} = 16.2°F \quad \text{(eq 3-45)}$$

$$v_o = 35.3 - 32.0 = 3.3°F \quad \text{(eq 3-46)}$$

$$\alpha = \frac{3.3}{16.2} = 0.20 \quad \text{(eq 3-47)}$$

b. Trial 1. Use a 2-inch layer of insulation and a 6-inch concrete leveling course.

—Pavement section: 14 inches of concrete
2 inches of insulation
6 inches of concrete leveling course

22 inches total

$$R_p = \frac{d}{K} = \frac{14}{12 \times 1.0} + \frac{2}{12 \times 0.024} + \frac{6}{12 \times 1.0}$$

$$= 8.60 \text{ hr ft}^2 \text{ °F/Btu} \quad \text{(eq 3-48)}$$

$$C_p = \frac{(14 \times 30) + (2 \times 0.28) + (6 \times 30)}{22}$$

$$= 27.3 \text{ Btu/ft}^3 \text{ °F} \quad \text{(eq 3-49)}$$

The calculation appears in table 3-3 and indicates that this pavement section has an excess of (2752 - 3033 =) 481 degree-days to prevent frost penetration into the silt subgrade.

c. Trial 2. Use a 1.5-inch layer of insulation and a 6-inch concrete leveling course.

—Pavement section:
14 inches of concrete
1.5 inches of insulation
6 inches of concrete leveling course

21.5 inches total

$$R_p = \frac{d}{K} = \frac{14}{12 \times 1.0} + \frac{1.5}{12 \times 0.024} + \frac{6}{12 \times 1.0}$$

$$= 6.86 \text{ hr ft}^2 \text{ °F/Btu} \qquad \text{(eq 3-50)}$$

$$C_p = \frac{(14 \times 30) + (1.5 \times 0.28) + (6 \times 30)}{21.5}$$

$$= 27.9 \text{ Btu/ft}^3 \text{ °F} \qquad \text{(eq 3-51)}$$

The calculation (see table 3-4) indicates that this pavement section will not prevent frost penetration into the silt subgrade as (2752 - 2553 =) 199 degree-days remain for subgrade penetration. The 2-inch thickness of insulation is therefore required.

Table 3-3. Insulated pavement design, no frost penetration.

Table 3-4. Insulated pavement design, frost penetration.

CHAPTER 4

TWO-DIMENSIONAL RADIAL HEAT FLOW

4-1. General.

Radial flow of heat is considered in thermal problems involving the design of pile foundations in permafrost (TM 5-852-4/AFM 88-19, Chapter 4), the construction of utility supply lines for the transport of water and sewage in permafrost areas and seasonal frost areas (TM 5-852-5/AFR 88-19, Volume 5), and the design of artificially frozen ground for retaining structures during construction. A number of the basic concepts and techniques used to calculate radial heat flow from cylindrical surfaces are discussed below.

a. Thermal Resistance. In analyzing heat flow for areas with cylindrical cross sections, the effective thickness for radial flow from a unit length of the cylinder is

$$\frac{1}{2\pi} \ln \frac{r_2}{r_1} \quad \text{or } 0.367 \log \frac{r_2}{r_1}$$

where

r_1 = inside wall radius (ft)

r_2 = outside wall radius (ft).

The thermal resistance R is equal to the effective thickness divided by the conductivity of the material between the two radii. As an *example*, a concrete conduit with a wall thickness of 6 inches and an inside diameter of 10 feet is surrounded by 4 inches of cellular glass insulation and 4 feet of dry gravel. Calculate the thermal resistance between the inside concrete wall and the outer edge of the gravel material. The following thermal conductivities are given:

— Concrete, K = 1.00 Btu/ft hr °F.

— Insulation, K = 0.033 Btu/ft hr °F.

— Gravel, K = 1.5 Btu/ft hr °F.

Let (see fig. 4-1 for values of r_1 - r_4)

r_1 = radius to inner wall of conduit

r_2 = radius to outer wall of concrete

r_3 = radius to outer edge of insulation

r_4 = radius to outer edge of gravel

$K_{1\text{-}2}$ = thermal conductivity of concrete

$K_{2\text{-}3}$ = thermal conductivity of insulation

$K_{3\text{-}4}$ = thermal conductivity of gravel.

$$R = 0.367 \left(\frac{1}{k_{1\text{-}2}} \log \frac{r_2}{r_1} + \frac{1}{K_{2\text{-}3}} \log \frac{r_3}{r_2} + \frac{1}{K_{3\text{-}4}} \log \frac{r_4}{r_3} \right) \qquad \text{(eq 4-1)}$$

$$= 0.367 \left(\frac{1}{1.0} \log \frac{5.5}{5.0} + \frac{1}{0.033} \log \frac{5.83}{5.5} + \frac{1}{1.5} \log \frac{9.83}{5.83} \right)$$

$$= 0.367\,(0.041 + 0.767 + 0.151) = 0.352 \text{ ft.}^2 \text{ hr °F/Btu.}$$

If the temperature in the conduit were 45°F and the temperature at the outer face of the gravel were 35°F, the heat flow per linear foot of conduit would equal

$$\frac{1}{0.352}\,(45 - 35) = 28.4 \text{ Btu/hr.} \qquad \text{(eq 4-2)}$$

b. Temperature field surrounding a cylinder. The sudden or step change in surface temperature discussed for semi-infinite slabs in paragraph 3-6*a* has application to heat-flow problems associated with pile foudations in permafrost. A mathematical solution is available for the problem where the surface temperature of a cylinder is suddenly changed from the uniform temperature of the surrounding medium, as long as there is no phase change. Figure 4-2 is used to determine the temperature T at a distance r from the center of a cylinder of radius r_1, at a time t after the surface temperature of the cylinder is changed from T_o to T_s. The temperature T_o represents the uniform temperature of the medium prior to the sudden change in surface temperature.

Figure 4-1. Illustration for example in paragraph 4-1a.

4-2. Pile installation in permafrost.

At many arctic and subarctic sites, pile foundations are commonly placed in preaugered holes, and the annular space between the oversized hole and pile is backfilled with a slurry of soil and water. The tangential adfreeze strength of the pile is principally a function of the bond developed between the pile and the frozen slurry. Dissipation of the sensible and latent heat of the slurry into permafrost is a major design factor because construction scheduling depends upon the time required for slurry freeze-back. Pile spacing is important as each pile adds heat, i.e., piles spaced too closely may increase permafrost temperatures, with a reduction of pile adfreeze strength and an increase in freeze-back time. This is particularly true in relatively warm permafrost (above 30°F). The following discussion assumes first that the slurry will freeze back naturally because of heat transfer between the surrounding permafrost and the slurry, second that the time required for freeze-back at a particular depth is predominantly influenced by the permafrost temperature at that depth, and third that the permafrost does not thaw. Under certain conditions, artificial refrigeration may be required to ensure freeze-back within a reasonable time. The volumetric specific heat of the slurry and the effect of vertical heat flow are assumed to have a negligible effect in computing required freeze-back time. With proper pile spacing, the slurry temperature reaches that of the surrounding permafrost in time. Surrounding temperatures, natural freeze-back time, proper pile spacing, and heat removal by refrigeration, as well as heat transfer by thermal piles, are discussed below.

a. Surrounding temperatures. The increase in permafrost temperatures during slurry freeze-back is determined using the technique described in paragraph 4-1*b*.. For *example*, a preaugered hole for a pile installation is 16 inches in diameter and the slurry is placed at a temperature of 32°F. The surrounding permafrost has a thermal diffusivity of 0.06 ft²/hr and an initial temperature of 28°F. Calculate the ground temperature at a distance of 3 feet from the center of the pile after an elapsed time of 48 hours. Given

r_1 = 0.667 ft	t = 48 hr
r = 3.00 ft	T_o = 28°F
a = 0.06 ft²/hr	T_s = 32°F

Figure 4-2. Temperature around a cylinder having received a step change in temperature.

therefore,

$$\frac{r_1}{\sqrt{at}} = \frac{0.667}{\sqrt{(0.06)(48)}} = 0.39 \qquad \text{(eq 4.3)}$$

$$\frac{r}{r_1} = \frac{3.00}{0.667} = 4.50. \qquad \text{(eq 4-4)}$$

From figure 4-2,

$$\frac{T - T_o}{T_s - T_o} = 0.18 \qquad \text{(eq 4-5)}$$

$$T = 0.18\,(T_s - T_o) + T_o = 0.18\,(32-28)+28$$

$$= 28.7°F. \qquad \text{(eq 4-6)}$$

This technique is also used to predict the increase in permafrost temperature during slurry freeze-back. Since it assumes a constant surface temperature for a cylinder, it is applicable only to the time of freeze-back. After the slurry has frozen, the permafrost temperature decreases and the model is not valid.

b. Natural freeze-back time. This heat transfer problem assumes a slurried pile to be a finite heat source inside a semi-infinite medium, with a suddenly applied constant temperature source (32°F) that dissipates radially into frozen ground of known initial temperature. The general solution for the natural freeze-back problem, based upon the latent heat content of the slurry, is shown in figure 4-3.

(1) For *example*, calculate the time required to freeze back a 12.5-inch diameter timber pile placed in an 18-

Figure 4-3. General solution of slurry freeze-back.

inch hole preaugered in permafrost and backfilled with a slurry for the following conditions:

— Permafrost:	Silty sand	
	Initial temperature = 27°F	
	Dry unit weight = 94 lb/ft^3	
	Water content = 25%.	
— Slurry backfill:	Silt, water	
	Placement temperature = 33.5°F	
	Dry unit weight = 72 lb/ft^3	
	Water content = 45%.	

In figure 2-3, the thermal conductivity of the permafrost is determined to be 1.1 Btu/ft hr°F. The volumetric heat capacity is calculated to be

$$[94\,(0.17 + 0.5\,\frac{25}{100}\,)] = 27.7\ \text{Btu/ft}^3\,°\text{F} \qquad \text{(eq 4-7)}$$

and the thermal diffusivity to be

$$\frac{1.1}{27.7} = 0.0397\ \text{ft}^2/\text{hr}. \qquad \text{(eq 4-8)}$$

The volumetric latent heat of the slurry is

$$(144 \times 72 \times 0.45) = 4670\ \text{Btu/ft}^3\,°\text{F} \qquad \text{(eq 4-9)}$$

and the latent heat per linear foot of backfill is

$$[\,\frac{\pi}{4}\,(1.5^2 - 1.04^2)]\ 4670 = 4280\ \text{Btu}. \qquad \text{(eq 4-10)}$$

When figure 4-3 is entered with a value of

$$\frac{Q}{Cr_2^2\Delta t} = [\,\frac{4280}{(27.7)(0.75^2)(5)}\,] = 55 \qquad \text{(eq 4-11)}$$

then,

$$\frac{at}{r_2^2} = 12.4. \qquad \text{(eq 4-12)}$$

The time required to freeze back the slurry backfill is

$$\frac{12.4 \times 0.75^2}{0.0397} = 176 \text{ hours or about } 7.3 \text{ days.} \qquad \text{(eq 4-13)}$$

At this time the slurry temperature is 32°F. Subsequent to freeze-back, the temperature of the slurry will continue to decrease and will approach the permafrost temperature. Ninety percent of the temperature difference will disappear in about twice the time required for freeze-back to 32°F. In this example, after a period of [7.3 + (2 × 7.3) =] 22 days, the slurry temperature would be approximately [32 - 0.90 (32 - 27) =] 27.5°F. The time (22 days) should be increased by 50 percent to permit an element of safety in the design. (Note: the sensible heat introduced by the pile and slurry was negligible in comparison to the latent heat introduced by the slurry and was not considered in calculations.)

(2) Permafrost temperature variations with depth, as discussed in TM 5-852-4/AFM 88-19, Chap. 4, should be considered in calculating freeze-back time. Figure 4-4 illustrates the effect of permafrost temperature on freeze-back for the above example. Since heat input is governed principally by the latent heat of slurry backfill, which is a function of slurry volume, moisture content and dry unit weight, a family of curves relating volumetric latent heat of slurry, permafrost temperatures and freeze-back time may be developed for a specific site to account

Figure 4-4. Specific solution of slurry freeze-back for varying pile shapes and preaugered hole diameters. To minimize the heat introduced by the slurry, the water content should be the minimum required for complete saturation. This can be best accomplished by backfilling with the highest dry unit weight material that can be processed and placed, i.e., a well-graded concrete sand with a 6-inch slump.

c. Pile Spacing. The effect of pile spacing on the overall rise of permafrost temperature resulting from installation of piles in preaugered holes is found by equating the latent heat of slurry backfill with the allowable sensible heat (temperature) rise of the surrounding permafrost. For *example*, calculate the minimum allowable pile spacing in the preceding example so that the permafrost temperature will not rise above 31°F. The following equation, equating the latent heat of the slurry to the change of sensible heat in a permafrost prism of side S, is used to determine the pile spacing:

$$S = \sqrt{(\pi r_2^2) + \frac{Q}{C \Delta T}} \qquad \text{(eq 4-14)}$$

where

S = grid pile spacing (ft)

r_2 = radius of augered hole (ft)

Q = latent heat of slurry per lineal foot (Btu/ft)

C = volumetric heat capacity of permafrost (Btu/ft³ °F)

ΔT = temperature rise of permafrost (°F).

Substitution of appropriate values from the above example and a maximum allowable permafrost temperature rise ΔT of 4°F give a minimum pile spacing S of

$$\sqrt{(3.14)(0.75)^2 + \frac{4280}{(27.7)(4)}} = 6.4 \text{ ft.} \qquad \text{(eq 4-15)}$$

This spacing may not keep local temperature from rising to more than 31°F; however, it will keep the entire mass of permafrost from reaching that temperature.

(1) Numerical analysis of a number of pile installations indicates that pile spacing should be *at least* five diameters of the drill hole size. A plot, similar to that shown in figure 4-4, may be prepared to relate pile spacing and permafrost temperature rise for the volumetric latent heat of the slurry backfill introduced into the drill hole. A family of curves may be developed to account for variation of slurry volumetric latent heat.

(2) In this example the slurry backfill was placed at a temperature slightly above freezing (33.5°F) and, theoretically, the sensible heat of the slurry should be considered. The volumetric capacity of the unfrozen slurry was [72(0.17 + 1.0 $\times$ 0.45) =] 44.6 Btu/ft³ °F, and with a temperature difference of (33.5 - 32 =) 1.5°F, this represents a sensible heat of (1.5 $\times$ 44.6 =) 67 Btu/ft³. A comparison of this quantity with the volumetric latent heat of the slurry (4670 Btu/ft³) shows that its heat may be considered negligible, as long as it near the freezing point.

d. Artificial freeze-back time. If permafrost is temperatures are marginal, it may be necessary to refrigerate the pile to accelerate slurry freeze-back time and to have refrigeration available if permafrost temperatures rise after construction. The following *example* shows calculations required to determine the amount of heat to be extracted from the ground. The average volume of slurry backfill for a group of piles is 31 cubic feet each. The slurry is placed at a n average temperature of 48°F and must be frozen to 23°F. A silt-water slurry of 80 lb/ft³ dry weight and 40 percent water content is used as backfill material, and an available refrigeration unit is capable of removing 225,000 Btu/hr. Calculate the length of time required to freeze back a cluster of 20 piles.

—Volumetric latent heat of backfill:

$$L = (144 \times 80 \times 0.40) = 4600 \text{ Btu/ft}^3. \qquad \text{(eq 4-16)}$$

—Volumetric heat capacity of frozen backfill:

$$C_f = 80\ [0.17 + (0.5 \times 0.4)] = 29.6 \text{ Btu/ft}^3\ °\text{F}. \qquad \text{(eq 4-17)}$$

—Volumetric heat capacity of unfrozen backfill:

$$C_u = 80\ [0.17 + (1.0 \times 0.4)] = 45.6 \text{ Btu/ft}^3\ °\text{F}. \qquad \text{(eq 4-18)}$$

—Heat required to lower the slurry temperature to the freezing point:

$$45.6 \times 31\ (48 - 32) = 22{,}618 \text{ Btu/pile.} \qquad \text{(eq 4-19)}$$

—Heat required to freeze slurry:

$$31 \times 4600 = 142{,}600 \text{ Btu/pile.} \qquad \text{(eq 4-20)}$$

—Heat required to lower the slurry temperature from the freezing point to 23°F;

$$29.6 \times 31\,(32 - 23) = 8258 \text{ Btu/pile.} \qquad \text{(eq 4-21)}$$

—Total heat to be removed from the slurry:

$$20\,(22{,}600 + 142{,}700 + 8200) = 3{,}470{,}000 \text{ Btu.} \qquad \text{(eq 4-22)}$$

—Time required for artificial freeze-back, excluding allowances for system losses:

$$3{,}470{,}000/225{,}000 = 15.5 \text{ hours.} \qquad \text{(eq 4-23)}$$

e. Heat transfer by thermal piles. Artificial freeze-back may be accomplished also by use of two types of self-refrigerated heat exchangers: a single-phase liquid-convection heat transfer device and a two-phase boiling-liquid and vapor convection heat transfer device. TM 5-852-4/AFM 88-19, Chapter 4 presents heat transfer rates for the two-phase system. There are few heat transfer field data available for the single-phase system.

4-3. Utility distribution systems in frozen ground.

General considerations for the design of utility systems in cold regions are given in TM 5-852-5/AFR 88-19, Volume 5.

a. Burying water pipes in frozen ground. Water pipes that are buried in frozen ground may be kept from freezing by any one of the following methods: 1) placing the water line in an insulated utilidor, which is a continuous closed conduit with all lines, such as water, sewage and steamlines, installed away from direct contact with frozen ground, 2) providing a sufficient flow velocity such that the water temperature at the terminus of the pipeline does not reach the freezing point or 3) heating the water at the intake or at intermediate stations along the line. A layer of insulation around a pipeline will retard, but not prevent, freezing of standing water in a pipe. The thermal analysis of a pipeline buried in frozen ground is complicated by the changing thermal properties, ice content, seasonal and diurnal changes of temperature and the intermittent water flow. Some calculation techniques applicable to the problem of buried utilities are presented below for standing and for flowing water. Additional techniques are presented in TM 5-852-5/AFR 88-19, Volume 5.

b. Freezing of standing water in a buried pipe. Problems with freezeup of stationary water must take into account the initial time required to lower the water temperature to the freezing point and the amount of time required to form an annulus of ice in the pipe. In most instances the danger point is reached when water begins to freeze.

(1) The time required to lower the temperature of nonflowing water in an insulated pipe to the freezing point is given by the equation

$$t = \frac{31.2}{K_i}\left(r_p^2 \ln \frac{r_i}{r_p}\right) \ln \frac{T_w - T_s}{32 - T_s} \qquad \text{(eq 4-24)}$$

where

- t = time (hr)
- K_i = thermal conductivity of insulation (Btu/ft hr °F)
- r_p = radius of pipe (ft)
- r_i = radius to outer edge of insulation (ft)
- T_w = initial water temperature (°F)
- T_s = temperature of surrounding soil (°F).

For *example*, a 12-inch diameter iron pipe containing water at 42°F is buried in 28°F soil. Determine the time required to lower the water temperature to 32°F if the pipe is insulated with 3 inches of cellular glass (K_i = 0.033 Btu/ft hr°F).

$$t = \frac{31.2}{0.033}\left(0.50^2 \ln \frac{0.75}{0.50}\right) \ln \frac{42-28}{32-28} = 120 \text{ hours (5 days)} \qquad \text{eq 4-25}$$

(2) Once the water temperature has been lowered to the freezing point, ice begins to form in an annular ring inside the pipe. The following assumptions are made to solve this problem:

—The water is initially at 32°F.

—The heat released by the freezing of water does not affect the surrounding ground temperatures.

—The volumetric heat capacity of the ice may be ignored.

—The thermal resistance of the pipe wall is negligible.

The solution predicts the time required to form an annulus of ice around the inner wall of the pipe. Knowledge of pipe radius, insulation thickness and

and thermal properties, thermal conductivity of ice, latent heat of fusion of water, and surrounding ground temperatures are necessary to solve this problem. The temperature of ground surrounding the pipe and the time during which the ground remains below freezing is difficult to estimate. The relationship between time and the radius of ice formed inside an insulated pipe is given by the expression

$$t = \frac{L r_p^2}{2 K \Delta T} \left[\left(\frac{K}{K_i} \ln \frac{r_i}{r_p} + 1/2 \right) \left(1 - \frac{r^2}{r_p^2}\right) - \left(\frac{r}{r_p}\right)^2 \ln \frac{r_p}{r} \right]. \quad \text{(eq 4-26)}$$

where

- t = time (hr)
- L = latent heat of water (9000 Btu/ft^3)
- r_p = radius of pipe (ft)
- K = thermal conductivity of ice (1.33 Btu/ft hr °F)
- ΔT = temperature difference between water and surrounding soil (°F, assume water temperature is 32°F)
- K_i = thermal conductivity of insulation (Btu/ft hr °F)
- r_i = radius to outer edge of insulation (ft)
- r = inner radius of ice annulus (ft).

If the pipe is not protected by insulation, the equation is

$$t = \frac{L r_p^2}{2 K \Delta T} \left[1/2 \left(1 - \frac{r^2}{r_p^2}\right) - \left(\frac{r}{r_p}\right)^2 \ln \frac{r_p}{r} \right]. \quad \text{(eq 4-27)}$$

This expression for an insulated pipe may be simplified by rearrangement and substitution of numerical values for the latent heat and the thermal conductivity of ice. This yields

$$t = 1690 \frac{r_p^2}{\Delta T} \{y\} \quad \text{(eq 4-28)}$$

where

$$y = \left[1 - \frac{r^2}{r_p^2} \left(1 - \ln \frac{r^2}{r_p^2}\right)\right]. \quad \text{(eq 4-29)}$$

The relationship between y and r/r_p is given in figure 4-5.

Following is an *example*. A 12-inch iron pipe, insulated with 3 inches of cellular glass (K_i = 0.033 Btu/ft hr °F), is placed in 28°F soil. Calculate the time required to reduce the bore of the pipe to 6 inches and the time required to completely freeze the water. Assume the rate of flow does not influence freezing. The time required to reduce the bore to 6 inches will be

$$t = \frac{9000\,(0.5)^2}{2(1.33)(32-38)} \left[\left(\frac{1.33}{0.033} \ln \frac{0.75}{0.50} + 0.5 \right) \left(1 - \frac{0.25^2}{0.50^2}\right) - \left(\frac{0.25^2}{0.50}\right) \ln \frac{0.50}{0.25} \right]$$

$$= 2640 \text{ hours.} \quad \text{(eq 4-30)}$$

The time required to completely freeze the water in the pipe will be

$$t = \frac{9000\,(0.5)^2}{2(1.33)(32-38)} \left[\left(\frac{1.33}{0.033} \ln \frac{0.75}{0.50} + 0.5 \right) \left(1 - \frac{0}{0.50^2}\right) - \left(\frac{0}{0.50}\right)^2 \ln \frac{0.50}{0} \right]$$

$$= 3550 \text{ hours.} \quad \text{(eq 4-31)}$$

The calculation is simplified since the term "r" for the inner radius of the pipe goes to zero. For an uninsulated pipe, the calculations assume the simplified form of

$$t = \frac{(1690)(0.5)^2}{(32-28)} \{y\} \quad \text{(eq 4-32)}$$

where r/r_p = 0 and y = 1.0 (fig. 4-5). Therefore,

t = 106 hours.

This example illustrates the effectiveness of insulation in retarding the freezeup of water in pipes, but as stated above, the assumptions used to develop these equations are conservative and the actual length of freezing time would be greater.

c. Thawing of frozen soil around a suddenly warmed pipe. In the preceding example it was assumed that water initially at 32°F was placed in frozen ground and the time relationship for freezing of the water in the pipe was determined. If the water was maintained above freezing, the frozen soil surrounding the ppe would thaw. To formulate a mathematical expression relating the time with the radius of thaw, it is assumed that: 1) the volumetric heat capacity of the soil is negligible, 2) both the surrounding soil and pipe are initially at 32°F, and 3) the pipe temperature is suddenly raised to a temperature above 32°F. The formula for an insulated pipe is

Figure 4-5. Freezeup of stationary water in an uninsulated pipe.

$$t = \frac{L r^2}{2 K_u \Delta T} \left[\left(\frac{K_u}{K_i} \ln \frac{r_i}{r_p} - 0.5 \right) \left[1 - \frac{r_i^2}{r^2} \right) + \ln \frac{r}{r_i} \right] \quad \text{(eq 4-33)}$$

where

- t = time (hr)
- L = latent heat of soil (Btu/ft^3)
- r = radius to outer edge of thawing soil (ft)
- K_u = thermal conductivity of unfrozen soil (Btu/ft hr °F)
- ΔT = temperature difference between pipe and surrounding soil (°F, assume soil at 32°F)
- K_i = thermal conductivity of insulation (Btu/ft hr °F)
- r_i = radius to outer edge of insulation (ft)
- r_p = radius of pipe (ft).

If the pipe is not protected by insulation, the expression is

$$t = \frac{L\,r^2}{2\,K_u\,\Delta T}\left[-0.5\left(1 - \frac{r_p^2}{r_2}\right) + \ln\frac{r}{r_p}\right]. \quad \text{(eq 4-34)}$$

For *example*, a 7- by 7-foot concrete utilidor has 9-inch concrete walls with an outer covering of 6 inches of insulation (K_i = 0.033 Btu/ft hr °F). The frozen soil around the utilidor is a sandy gravel with a dry density of 115 lb/ft³ and a water content of 7.8 percent at a temperature of 32°F. Neglect the thermal resistance of the concrete, and determine the time required to thaw 1 foot of soil when the temperature of the utilidor walls is suddenly raised to 50°F.

K_u = 1.2 Btu/ft hr °F
L = 1290 Btu/ft^3
K_i = 0.033 Btu/ft hr °F.

For calculation, square sections may be treated as cylinders of the same perimeter.

Symbol	Dimension-square (ft)	Equivalent radius (ft)
r	9.5 + 24/12 = 11.5	7.33
r_i	8.5 + 12/12 = 9.5	6.04
r_p	7.0 + 18/12 = 8.5	5.42

$$t = \frac{(1290)(7.33)^2}{2(1.2)(50-32)}\left[\left(\frac{1.2}{0.033}\ln\frac{6.04}{5.42} - 0.5\right)\left(1 - \frac{6.04^2}{7.33^2}\right) + \ln\frac{7.33}{6.04}\right]$$

$$= 2060 \text{ hours.} \quad \text{(eq 4-35)}$$

d. Practical considerations.

(1) The above-mentioned formulas indicate the relationship between time, radius of freeze or thaw, and temperature difference between the water in the pipe and the ground. In sufficient time, standing water in the pipe will freeze or frozen ground will thaw, depending on temperature differentials. For practical problems the assumed constant temperature differentials will not exist for a long time but will vary with season and even with the hour at shallow depths. The freezing and thawing index concept considers the intensity of temperature differential from freezing (32°F) and the duration of this differential. In the preceding equations, the time t can be multiplied by the temperature differential ΔT to give either a freezing or thawing index, and the radius of ice formation or thawed ground radius can be determined by trial-and-error. It was shown that a temperature differential of (32-28) = 4°F lasting for 3150 hours would result in complete freezeup of the water in the pipe; this is equivalent to a freezing index of [(4 × 3150)/24 =] 524 degree-days. If the freezing index at the depth of pipe burial were less than 524, the standing water in the pipe would not completely freeze in that time. Thus, the freezing index at a particular depth can be used to forecast freezeup of stationary water in pipes located in the annual frost zone.

(2) Even insulated water lines located in frozen ground usually require an inlet water temperature significantly above freezing. Whether the water lines are insulated or not, this may thaw some of the surrounding frozen ground. This thawed annulus will retard water freezeup in the pipe if and when flow conditions change. The situation is practice is generally complicated by the intermittent character of water demand. In some northern communities the problems of irregular water demand are solved by constructing the water lines in a continuous loop with provisions for periodic flow reversals. Water temperatures should be closely monitored and water usage patterns considered in estimating water freezeup.

e. Freezing and flowing water in buried pipes. Problems involving freezing of flowing water in buried pipes require knowledge of the distance the water will flow before the temperature of the water lowers to the freezing point. By providing enough above-freezing water, the loss of heat to the surrounding frozen soil can be balanced to provide an outlet temperature slightly above freezing. The problems of freezing of flowing water in insulated and bare pipe are illustrated below.

(1) *Insulated pipe.* It is assumed that 1) the temperature of the frozen ground surrounding the pipe is constant for the period of flow over the

entire length of pipe, 2) the effect of friction heat developed by water flow is negligible, 3) the thermal resistance and heat capacity of the pipe wall are negligible, and 4) the temperature distribution of the water in the pipe is uniform at each cross-sectional area. The velocity required to prevent freeze-up of flowing water in a pipe is given by

$$V = \frac{s K_i}{112{,}000\, r_p^2 (\ln \frac{r_i}{r_p})(\ln \frac{T_1 - T_S}{T_2 - T_S})} \quad \text{(eq 4-36)}$$

where

- V = velocity of flow (ft/s)
- s = length of pipeline (ft)
- K_i = thermal conductivity of insulation (Btu/ft hr °F)
- r_p = radius of pipe (ft)
- r_i = radius to outer edge of insulation (ft)
- T_1 = inlet water temperature (°F)
- T_2 = outlet water temperature (°F)
- T_S = temperature of surrounding frozen soil (°F).

For *example*, an 11,000-ft long, 6-inch-diameter pipe is buried in 10°F soil. The pipe is covered with a 2-inch layer of insulation (K_i = 0.03 Btu/ft hr °F) and the inlet water temperature is 39°F. Calculate the velocity of flow required to keep the water from freezing.

$$V = \frac{(11{,}000)(0.03)}{112{,}000\,(0.25)^2 (\ln \frac{0.417}{0.25})(\ln \frac{39 - 10}{32 - 10})}$$

$$= 0.33 \text{ ft/s (20 ft/min).} \quad \text{(eq 4-37)}$$

To provide for temporary reductions in flow and in recognition of the uncertainties concerning the manner of ice formation within the pipe, it is recommended that the velocity of flow be doubled in design.

(2) Uninsulated pipe.

(a) When flowing water is first introduced into a bare pipe buried in frozen ground, the heat loss from water is greater than it is after the system has been in operation for a period of time. The initial heat loss is greater because the pipe wall and the soil immediately adjacent to the pipe are colder than they are after water has flowed over a time. Soil temperatures surrounding the pipe increase and eventually become reasonably stable with time. An expression relating these variables is

$$\frac{T_1 - T_S}{T_2 - T_S} = \exp \frac{s}{2r_p} \times \frac{h}{V} \times \frac{1}{5.6\times 10^4} \quad \text{(eq 4-38)}$$

where

- T_1 = inlet water temperature (°F)
- T_S = frozen soil temperature (°F)
- T_2 = outlet water temperature (°F)
- s = length of pipeline (ft)
- $2r_p$ = diameter of pipe (ft)
- h = heat transfer coefficient (Btu/ft^2 hr °F)
- V = velocity of flow (ft/s).

A nomogram of this equation is shown in figure 4-6.

(b) Limited field experiments in clay and sandy clay soils suggest values of h for metal pipelines subject to normal use (conditions or intermittent flow) of 6.0 for the initial period of operation an 2.0 thereafter. These values are not applicable for pipes smaller than 4-inches in diameter. The h value is dependent upon the thermal properties of the surrounding soil, the diameter of the pipe, the type of pipe material and the temperature gradient in the ground around the pipe's radius. The value for h given above provides a reasonable basis for design of pipelines in which the total quantity of water consumed per day is at least eight times the volume of pipes in the entire system. The time of operation required for the temperature distribution in the water to stabilize is approximately.

$$t_o = 0.005 \frac{s}{V} \quad \text{(eq 4-39)}$$

where

- t_o = time (hr)
- s = length of pipeline (ft)
- V = velocity of flow (ft/s).

(c) For *example*, water at an inlet temperature of 40°F flows at 2 ft/s in a 12-inch iron pipeline, 2.2 miles long. The ground temperature surrounding the pipe is 25°F. Estimate the outlet water temperature during the initial period of flow (h = 6.0) and after surrounding temperatures have stabilized (h = 2.0).

—Initial period (see fig. 4-6):

$$\frac{s}{2r_p} = \frac{2.2 \times 5280}{1} = 1.16 \times 10^4 \qquad \text{(eq 4-40)}$$

V = 2 ft/s.

h = 6.0 Btu/ft^2 hr °F

thus

$$\frac{T_1 - T_S}{T_2 - T_S} = 1.77 = \frac{40 - 25}{T_2 - 25} \qquad \text{(eq 4-41)}$$

$T_2 = 33.5$°F.

—Stabilized period:

$$\frac{s}{2r_p} = 1.16 \times 10^4$$

V = 2 ft/s

h = 2.0 Btu/ft^2 hr °F

thus

$$\frac{T_1 - T_S}{T_2 - T_S} = 1.22 = \frac{.40 - 25}{T_2 - 25} \qquad \text{(eq 4-42)}$$

$T_2 = 37.3$°F.

Figure 4-6. Temperature drop of flowing water in a pipeline.

To operate on the safe side, the outlet temperature T_2 should remain at or above 35°F. The calculated initial water temperature of 33.5°F would be considered unsafe and either the flow velocity should be increased to approximately 3 ft/s or the inlet water temperature should be raised to about 43°F. These precautions would be needed only for an initial period, i.e.,

$$t_o = 0.005 \frac{S}{V} = 0.005 \frac{11,600}{2}$$

$$= \text{approximately 29 hours.} \qquad \text{(eq 4-43)}$$

f. Design considerations. The calculation techniques presented above indicate the principal factors to be considered in design of water distribution lines placed in frozen ground. The use of these techniques together with recognition of the complexities of actual in-pipe ice formation and sound engineering judgment provides a basis for design of pipelines in areas of seasonal frost and permafrost. Changing the surface cover over an installed pipeline will affect the distribution of temperatures with depth and may result in depressing the temperatures adjacent to the pipe. This is particularly true if a natural vegetative cover is stripped and replaced by a snow-free pavement. The influence of new construction above an existing pipeline may require a change in operating procedures for the system, such as an increase in the velocity or flow or additional heating at the inlet.

4-4. Discussion of multidimensional heat flow.

a. The relatively simple analytical techniques discussed in this manual are not always sufficient for considering the concurrent thermal effects of multidimensional temperature change and soil water phase transformation. The one-dimensional and radial heat-flow computation techniques presented in this manual were based on field observations and the use of reasonable simplifying assumptions. The techniques are intended to facilitate analysis and to promote adequate design. The assumptions involved and technique limitations have been emphasized.

b. Heat flow beneath heated structures is multidimensional because of the finite boundaries of such structures. The ground surface temperature adjacent to the south side of the building is generally higher than that on the north side, and the ground surface temperature on the west side is generally higher than on the east side; in addition to this influence, the three-dimensional temperature distribution beneath the building will be affected by the plan dimensions of the floor. Three-dimensional solutions are available to the problem of heat flow in homogeneous materials beneath the surface of a heated finite area surrounded by an infinite area subject to a dissimilar surface temperature condition; however, the solutions consider only the effects of temperature change and not the effects of phase transformations. Such solutions tend to be rather complex and unwieldy, and their neglect of latent heat generally results in an over-estimation of the depth of freeze or thaw. The magnitude of this over-estimation is dependent on the quantity of moisture in the frozen or thawed soil.

c. The example given in paragraph 3-8*a* for calculating the depth of thaw beneath a heated slab-on-grade building considered only one-dimensional vertical heat flow and excluded lateral heat flow from the soil beneath the building to the surrounding soil mass in the winter. The amount of lateral heat flow would depend on the building dimensions and the wintertime soil temperature gradient. Slab-on-grade, heated structures usually prevent frost penetration under the center of the building and result in a thaw bulb in the foundation soil that may cause permafrost degradation with time. This type of construction is discussed in TM 5-852-4/AFM 88-19, Chapter 4.

APPENDIX A

Symbol	*Description*	*Units*
a	Thermal diffusivity	ft^2/day or ft^2/hr
a′	Shortest dimension	ft
A	Sinusoidal temperature amplitude	°F
A_d	Duct cross-sectional area	ft^2
c	Specific heat	Btu/lb °F
c_p	Specific heat of air at constant pressure	Btu/lb °F
C	Volumetric heat capacity	Btu/ft^3 °F
C_f	Volumetric heat capacity in frozen condition	Btu/ft^3 °F
C_u	Volumetric heat capacity in unfrozen condition	Btu/ft^3 °F
d	Thickness of soil layer	ft
D_e	Equivalent duct diameter	ft
e	Roughness factor	ft
erf	Error function; $\text{erf } z = (2/\sqrt{\pi}) \int_0^z e^{-\mu^2} d\mu$, where erf ∞ = 1 and erf (-z) = -erf z	dimensionless
exp(x)	e^x	dimensionless
f	Frequency of sine wave	cycles/day
F	Air freezing index	degree-days
f′	Friction factor	dimensionless
h	Heat transfer coefficient	Btu/ft^2 hr °F
h_{rc}	Surface conductance for combined radiation and convection	Btu/ft^2hr °F
h_d	Draft head	inches of water
h_f	Friction head	inches of water
h_v	Velocity head	inches of water
H	Stack height	ft
I	Air thawing index	degree-days
I_f	Floor thawing index	degree-days
k	Coefficient of thermal conductivity	Btu/ft^2 hr °F per in.
K	Thermal conductivity	Btu/ft hr °F
K_f	Thermal conductivity in frozen condition	Btu/ft hr °F
K_u	Thermal conductivity in unfrozen condition	Btu/ft hr °F
L	Volumetric latent heat of fusion	Btu/ft^3
m	Duct spacing	ft
ln	Natural logarithm	dimensionless
ℓ	Duct length	ft

Symbol	*Description*	*Units*
ℓ_b	Allowance for bends, etc., in duct	ft
ℓ_e	Equivalent duct length	ft
ℓ_s	Length of straight duct	ft
MAT	Mean Annual temperature	°F
n	"n"-factor = $\frac{\text{surface index}}{\text{air index}}$	dimensionless
nF	Surface freezing index	degree-days
nI	Surface thawing index	degree-days
N_R	Reynolds number	dimensionless
P	Period of sine wave	365 days
Q	Latent heat per linear foot of slurry backfill	Btu/ft
r	Radius	ft
R	Thermal resistance	ft^2 hr °F/Btu
S	Pile spacing	ft
s	Length of pipeline	ft
t	Time	hours or days
T	Temperature	°F
T_o	Initial temperature	°F
T_R	Temperature rise in duct air	°F
T_s	Surface temperature	°F
T_S	Soil temperature	°F
T_w	Water temperature	°F
V	Velocity of flow	ft/s
V_o	Initial temperature of the soil with respect to 32°F	°F
v_s	Average surface temperature with respect to 32°F	°F
w	Water content	percent dry weight
X	Depth of freeze or thaw	ft
α	Thermal ratio = v_o/v_s	dimensionless
γ_d	Dry unit weight	lb/ft^3
ε	Efficiency	percent
λ	Lambda coefficient	dimensionless
μ	Fusion parameter	dimensionless
ν	Kinematic viscosity	ft^2/hr
π	Pi	3.14
ρ	Density of air	lb/ft^3
ϕ	Heat flow to duct	Btu/ft^2 per hr

APPENDIX B

THERMAL MODELS FOR COMPUTING FREEZE AND THAW DEPTHS[1]

B-1. Analytical Solutions.

a. The Oxford University Press publication, *Conduction of Heat in Solids*,[2] presents solutions to many one-, two- and three-dimensional heat flow problems. Homogeneous isotropic materials are used in most solutions, but some solutions are presented for layered systems. U.S.G.S. Bulletin 1083-A[3] developed a one-dimensional technique for predicting the damping of a periodic surface temperature at different depths in a two- and three-layered soil system. Unidirectional heat flux was considered. U.S.G.S. Bulletin 1052-B[4] developed a method for estimating the three-dimensional thermal regime in a homogeneous isotropic soil beneath a heated structure. None of these techniques considers phase change of the soil moisture. Neglect of the effects of latent heat of fusion of the soil moisture normally does not cause substantial error in prediction of frost depths in soil of low water content. Differences between actual and computed thaw depths increase rapidly with increasing water content because of the increased volumetric heat capacity and greater latent heat of the wetter soil.

b. Several empriical and semi-empirical equations have been developed that consider latent heat of fusion of the soil. the Stefan equation was originally developed to calculate the thickness of ice on a calm body of water (isothermal at the freezing temperature, 32°F) expressed as:

$$X_i = \sqrt{48 K_i F / L_i} \qquad \text{(eq B-1)}$$

where

X_i = ice thickness (ft)

K_i = thermal conductivity of ice (Btu/ft hr °F)

F = freezing index (degree-days)

L_i = volumetric latent heat of fusion of ice (Btu/ft^3).

The Stefan equation has been modified by many individuals and agencies, and many similar equations have been developed. Some of the equations use functions or initial conditions slightly different from those used in the original Stefan model. The most widely used equation to estimate seasonal freeze and thaw depths is the modified Berggren equation. Application of this equation, given below, has been very widespread in North America. USACRREL Special Report 122[5] developed a computer program for calculating freeze and thaw depths in layered systems using this equation:

$$X = \sqrt{48 K n F / L} \quad \text{or} \quad X = \lambda \sqrt{48 K n I / L} \qquad \text{(eq B-2)}$$

where

X = depth of freeze or thaw (ft)

K = thermal conductivity of soil (Btu/ft hr °F)

L = volumetric latent heat of fusion (Btu/ft^3)

n = conversion factor from air index to surface index (dimensionless)

F = air freezing index (degree-days)

I = air thawing index (degree-days)

λ = coefficient that considers the effect of temperature changes within the soil mass. It is a function of the freezing (or thawing) index, the mean annual temperature and the thermal properties of the soils.

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1 The documents mentioned in this appendix are sources for additional information and are found in the bibliography.

2 Carslaw and Jaeger, 1959.

3 Lachenbruch, 1959.

4 Lachenbruch, 1957.

5 Aitken and Berg, 1968.

c. An equation very similar to the Stefan equation is used in the USSR to calculate the "standard" depth of freezing for foundation designs. Many other closed-form analytical techniques are also used in the USSR.

B-2. Graphical and analog methods.

a. Graphical methods have been used to estimate depths of freeze and thaw. The flow net technique can be used to estimate steady-state temperature conditions. National Research Council of Canada, Technical Paper No. 163,[6] presents a graphical means to determine temperature in the ground under and around natural and engineering structures lying directly on the ground surface.

b. Analog techniques are also used to estimate freeze and thaw depths. Table B-1 shows thermal, fluid and electrical analogies. Electric analog computers are available, cost relatively little and are reasonably simple to use. The primary disadvantage of these machines are that reprogramming is normally necessary for each problem and complex geometries are difficult to simulate adequately. Hydraulic analogs are also available. The primary disadvantages of these computers are their complex tubing systems, space requirements, and the necessity to thoroughly clean and reconstruct them for each problem. At any instant of time, however, hydraulic analogs graphically show the temperature distribution.

[6] Brown, 1963.

B-3. Numerical techniques.

a. Because of the widespread availability of electronic digital computers, their application to numerical solutions of the continuity equation is commonplace. Numerical procedures are approximations to the partial differential equation; however, they are much more accurate and versatile in solving complex transient heat flow problems than are the analytical techniques. Many computer programs allow flexible definition of boundary and initial conditions for both one- and two-dimensional problems. General background information on the *finite difference methods* available to solve heat flow problems are discussed in International Textbook Company's *Heat Transfer Calculations by Finite Differences.*[7] Since rectangular elements are normally used, complex geometries are difficult to simulate accurately unless small element sizes are used.

b. Use of the *finite element technique* is also widespread (see *The Finite Element Method in Engineering Science*[8] or *The Finite Element Method in Structural and Continuum Mechanics,*[9] both from McGraw-Hill). Elements of various shapes can be used with this technique; but the triangular shape is commonly used for two-dimensional problems. Complex boundary geometries can be more closely

[7] Dusinberre, 1961.
[8] Zienkiewicz, 1967.
[9] Zienkiewicz, 1971.

Table B-1. Thermal, fluid and electric analogs (U.S. Army Corps of Engineers).

Item		*Medium*					
		Thermal		*Fluid*		*Electric*	
A - Variables	(1)	Heat	μ	Volume	S	Charge density	ρ
	(2)	Heat flux	$\vec{q}$	Flow	$\vec{Q}$	Current density	$\vec{j}$
	(3)	Temperature	T	Head	H	Voltage	e
B - Principles:							
Continuity	(1)	$\frac{\partial \mu}{\partial t} + \vec{\nabla} \cdot \vec{q} = 0$		$\frac{\partial S}{\partial t} + \vec{\nabla} \cdot \vec{Q} = 0$		$\frac{\partial \rho}{\partial t} + \vec{\nabla} \cdot \vec{j} = 0$	
Conductivity	(2)	$\vec{q} = -k\vec{\nabla}T$		$\vec{Q} = -k\vec{\nabla}H$		$\vec{j} = -\sigma\vec{\nabla}e$	
Capacitance	(3)	$d\mu = CdT$		$dS = AdH$		$\rho dV = Cde$	

simulated using finite element procedures. For multidimensional heat flow problems, the finite element procedure is frequently more efficient, i.e., it requires less computer time than the finite difference technique.

c. Many flexible computer programs exist that simulate heat conduction and phase change in soils. Each has its own particular data requirements, computational capabilities, and acquisition costs and restraints. USACRREL has completed and documented a model that may be useful in solving many analytical problems related to construction in the Arctic and Subarctic; other models are under development. Contact HQ (DAEN-ECE-G) or HQ AFESC for assistance in selecting an appropriate model.

APPENDIX C

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TM 5-852-1/AFR 88-19, Volume 1	Arctic and Subarctic Construction, General Provisions
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